

# SOME EXPERIMENTS BEARING ON THE HYPOTHESIS THAT THE VISUAL SYSTEM ANALYSES SPATIAL PATTERNS IN INDEPENDENT BANDS OF SPATIAL FREQUENCY<sup>1</sup>

G. BRUCE HENNING,\* B. GEVENE HERTZ† and D. E. BROADBENT\*

\*Department of Experimental Psychology, South Parks Road, Oxford OX1 3UD, England; and

†The Biomedical Engineering Center, Northwestern University, Evanston, Illinois 60201, U.S.A.

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**Abstract**—Gratings with three sinusoidal components of high spatial frequency are shown to interact with a sinusoidal grating two octaves lower in frequency. This finding is inconsistent with the hypothesis that the visual system analyses spatial patterns in independent narrowly-tuned bands of spatial frequency.

## INTRODUCTION

Campbell and Robson (1968) have suggested that the visual system might act as if it were composed of many independent linear mechanisms, sometimes called "channels", each selectively sensitive to a limited range of spatial frequencies. They based their hypothesis on the finding that a grating with alternating black and white bars was detected at contrasts consistent with their observers' detecting only one of the spatial frequency components of which the grating was comprised. The particular component of the pattern to which the visual system was most sensitive depended on the contrast of the various components of the pattern and on the sensitivity of the visual system at the spatial frequency of each component. A great deal of evidence supporting Campbell and Robson's notion has accumulated; studies of the detectability of complex gratings (Graham and Nachmias, 1971; Sachs, Nachmias, and Robson, 1971), experiments showing that adaptation effects are confined to narrow bands of spatial frequency (Blakemore and Campbell, 1969), and simultaneous masking experiments (Carter and Henning, 1971; Stromeyer and Julesz, 1973; Henning and Hertz, 1974; Sansbury, 1974) all provide results consistent with Campbell and Robson's hypothesis.

The experiments reported here show reciprocal masking between patterns that have the same periodicity although they occupy bands of spatial frequency that are at least two octaves apart. These experiments are not consistent with the hypothesis that the visual system analyses spatial patterns in independent channels each sensitive only to a range of spatial frequencies an octave above and below the most sensitive frequency of the channel.

The stimuli we used were a low-frequency sinusoidal grating and a complex grating having three components of high spatial frequency.

It is important to realize that a grating can be periodic, that is, it can repeat in some spatial interval, without having a component at the spatial frequency corresponding to its period. For example, one of our complex gratings contained components at about 8, 10 and 12 c/deg. The three-component grating looks like a 10 c/deg grating with a contrast that varies across the visual field. The contrast in fact varies sinusoidally and the entire pattern repeats twice every degree. Thus the pattern has a period that corresponds to the low spatial frequency of 2 c/deg, even though the lowest spatial frequency of its three components is 8 c/deg.<sup>2</sup>

The distinction between the period of a waveform and the frequency region of its components is important in auditory psychophysics. In vision, however, the periodicity of a grating has not often been experimentally separated from the frequency of the components of the grating. Our experiment with simple sinusoidal gratings, in which spatial frequency and the reciprocal of the period are necessarily the same, give results consistent with Campbell and Robson's hypothesis—there is little interaction between sinusoidal gratings that differ by more than a factor of two in spatial frequency; on the other hand, our experiments with complex gratings show large interactions between gratings that are much farther apart in frequency.

## GENERAL PROCEDURE

Each of our experiments was a two-alternative forced-choice grating detection experiment. The signal grating was presented in one of two observation intervals each 1 sec in duration and separated by a 600-msec pause. The observers (the same two in all experiments)<sup>3</sup> were required to indicate in a subsequent 750-msec answer interval whether the first or second observation interval had contained the signal. The observers were then informed which interval had in fact contained the signal, and a new trial was begun. All intervals were clearly marked for the observers by bursts of audible noise delivered through earphones.

The probability that the signal was in the first interval was 0.5 on each trial and each trial lasted about 4 sec. After a block of 50 trials, the signal contrast (determined separately for each component of the stimulus as the ratio of the difference between the maximum and minimum luminance of the component to the sum of the maximum and minimum luminance of that component) was changed and a new block of 50 trials begun. In this way, we determined psychometric functions relating the percentage of correct responses to sig-

<sup>1</sup> This research was carried out while we were at the Applied Psychology Unit in Cambridge and one of us (GBH) was employed by the Canadian Defence and Civil Institute of Environmental medicine.

<sup>2</sup> The finite  $6^\circ$  extent of the gratings implies that all the components we describe as spectral lines have an effective bandwidth of at least 0.17 c/deg. We shall ignore this effect throughout.

<sup>3</sup> Two of the authors are the Observers (Os).

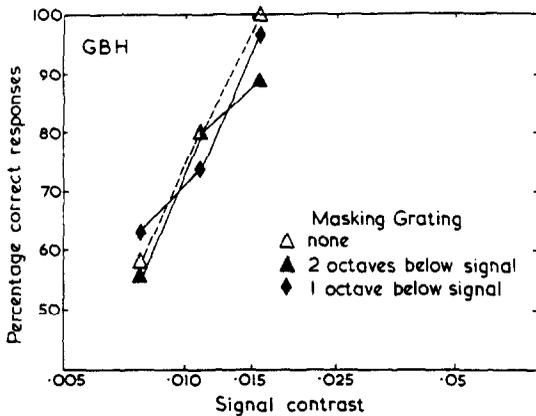


Fig. 1. This shows the percentage correct detection of a 7.6 c/deg signal grating as a function of its contrast. The open symbols represent the case in which no masking grating was used; the closed triangles, the case with a 1.9 c/deg masking grating of 17.4 per cent contrast was present; the lozenges, the case with a 3.8 c/deg grating of 17.4 per cent contrast. Each data point is based on 100 observations from observer GBH.

nal contrast. The detectability of a signal grating was measured in different conditions: against the mean luminance of the display or against various masking gratings. The masking gratings, when used, were present on the display during both observation intervals but not otherwise. The phase of the masking grating—the location of its white bars within the display—was fixed for each observation interval but varied haphazardly from interval to interval. The phase of the signal grating did not vary. Both the signal and masking gratings were turned on and off abruptly. Three types of signal and masking gratings were used: simple sinusoidal gratings with only one component, complex gratings comprised of three simple gratings and a “noise” grating comprised of a set of simple gratings with varying contrast.

All the gratings were generated in a Hewlett-Packard 1300 X-Y display using the technique of Campbell and Green (1965) with the equipment described by Carter and Henning (1971). The mean luminance of the display, 1.55 ft-L, was not altered by the addition of the signal grating, the masking grating, or both. A matte black frame was arranged to provide a single  $6^\circ \times 6^\circ$  square field in a black surround and all the gratings used were vertical. The observers, both with corrected vision, viewed the gratings monocularly. There was no fixation mark.

The signal and masking gratings in this and in subsequent experiments were generated using separate digital-to-analogue converters in association with the Modular One computer at MRC/APU. It is essential for the interpretation of our results to know that the level of harmonic distortion produced in the luminance pattern of the display was negligible. Using the technique described by Carter and Henning (1971), we confirmed that the second and third harmonics produced in the luminance pattern of the display by a sinusoidal signal were at least 37 dB below the level of the sinusoid; the luminance pattern was essentially sinusoidal provided the contrast of the signal was less than about 63 per cent. The stimuli in our experiments all had less than 63 per cent contrast.

#### PRELIMINARY EXPERIMENT

##### Procedure

In this experiment the signal was a 7.6 c/deg sinusoidal grating. Its detectability was measured with no masking grating, with a 1.9 c/deg masking grating of 17.4 per cent contrast, and with a 3.8 c/deg masking grating having 17.4 per cent contrast.

#### Results

Figures 1 and 2 show the percentage of correct responses as a function of the signal contrast for each observer. Each data point is based on at least 100 observations. Open symbols in the figures represent data from the condition in which no masking grating was present. Filled symbols represent the conditions in which a low-frequency masking grating was present. There is a measurable difference in the sensitivity of the observers, but, for both observers, the 1.9 c/deg masking grating two octaves below the spatial frequency of the signal produced no measurable effect. The 3.8 c/deg masking grating one octave below the signal frequency had no measurable effect on the detectability of the signal for Observer 1 and may possibly have produced a slight improvement in the performance of Observer 2. These results are consistent with those from other masking experiments (Stromeyer and Julesz, 1973; Sansbury, 1974; Henning and Hertz, 1974), from adaptation experiments (Blakemore and Campbell, 1969; Maudarboccos and Ruddock, 1973), and from experiments involving the detectability of multiple component gratings (Campbell and Robson, 1968; Graham and Nachmias, 1971).

The conclusion frequently drawn from such results is that luminance patterns comprised of sufficiently different spatial frequencies are processed separately in different “channels” each sensitive to a different band of spatial frequency.

The next experiment measured the effect of the 1.9 c/deg masking grating on a complex stimulus comprised of three components of high spatial frequencies none of which was lower in frequency than the 7.6 c/deg sinusoidal grating of the preliminary experiment.

#### EXPERIMENT 1

##### Procedure

The task faced by the same two Os in this experiment may be described in several ways.

A 9.5 c/deg sinusoidal grating was present in both observation intervals with a contrast of 6.3 per cent. This component may be called the carrier. In one of the observation intervals two sinusoidal gratings, equal in contrast, were added so that periodically in space the maximum luminance of each coincided with the carrier maximum. The two sinusoidal components, at 7.6 c/deg and 11.4 c/deg, will be called

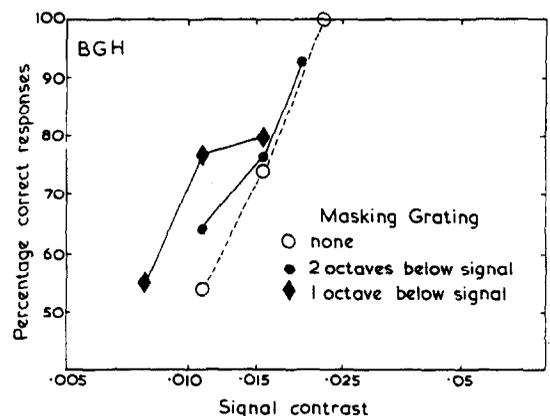


Fig. 2. This shows data for observer BGH from the conditions described in Fig. 1.

sidebands and constituted the signal that the *O*s attempted to detect. (The spatial frequencies of the sidebands and the carrier are all integer multiples of 1.9 c/deg. are thus harmonically related, and produced, in the signal interval, a periodic stimulus with a period of 1/1.9°.) The sidebands were kept equal in contrast but their contrast was varied relative to that of the carrier to determine the percentage correct detection of the sidebands as a function of their contrast. The two-alternative forced-choice technique used in the preliminary experiment was used again. Detection performance was measured in two conditions: against the uniform mean luminance of the display or against the 1.9 c/deg masking grating with 17.4 per cent contrast used in the preliminary experiment. The masking grating and the carrier grating were present only during the observation intervals.

A different description of the task involves noting that the high frequency pattern produced in the signal interval by adding the sidebands to the carrier so that the luminance peaks of all three components coincide periodically is equivalent to that produced by sinusoidally modulating<sup>4</sup> the contrast of the carrier. This may be seen by expanding the equation that describes the luminance of a contrast modulated pattern,  $L(s)$ , given by

$$L(s) = \bar{L}[1 + K(1 + m \cos 2\pi f_m s) \sin 2\pi f_c s], \quad (1)$$

where  $f_c$  and  $f_m$  are the carrier and modulation frequencies respectively,  $\bar{L}$  is the mean luminance,  $K$  is the contrast of the carrier, and  $m$ —the depth of modulation—determines the contrast of the sidebands relative to that of the carrier. In our experiment,  $f_c$  was 9.5 c/deg,  $f_m$  was 1.9 c/deg,  $\bar{L}$  was 1.55 ft-L, and  $K$ , the carrier contrast, was 6.3 per cent. The parameter  $m$  was zero in the observation interval containing no signal and had some positive value when the signal was added. Expanding equation (1) shows that

$$L(s) = \bar{L}\{K[(m/2) \sin 2\pi(f_c - f_m)s + \sin 2\pi f_c s + (m/2) \sin 2\pi(f_c + f_m)s] + 1\}. \quad (2)$$

Thus the three-component pattern described earlier is equivalent to the contrast modulated waveform of equation (1).

Figure 3 attempts to clarify the task faced by our observers. Figure 3(a) shows the luminance pattern produced by adding sidebands at frequencies  $kf_c$  and  $(k + 2)f_c$  to a carrier at  $(k + 1)f_c$  c/deg. The luminance pattern shown is for the case in which the depth of modulation is 100 per cent; i.e.  $m$  in the equation (2) is equal to one and each sideband has one-half the contrast of the carrier. On the right of Fig. 3(a) is shown the spectrum of the luminance pattern. The relative amplitude and frequency of the three component complex is shown in solid lines. There is no component at  $f_0$  c/deg, the frequency of modulation and the reciprocal of the spatial period of the pattern. The spectrum of the masking grating is indicated by the dotted line at spatial frequency  $f_0$ . Figure 3(b) shows the case in which no sidebands have been added. The luminance pattern is sinusoidal with a single component at the carrier frequency  $(k + 1)f_c$  c/deg.

The *O*s were required, then, to discriminate between gratings with luminance profiles like that shown in Fig. 3(a and b), first against the mean luminance of the display, and then against a sinusoidal masking grating of frequency  $f_0$  c/deg. The depth of modulation,  $m$ , was varied to determine the percentage of correct responses as a function of sideband contrast.

Results

Figure 4 shows the percentage of correct responses

<sup>4</sup> In writing about gratings some authors used the term "modulation" simply to describe a sinusoidal luminance pattern and readers should note the difference between that usage and ours. We use "modulation" to indicate a spatial variation in the contrast of a grating rather than the spatial variation in luminance that defines the grating itself.

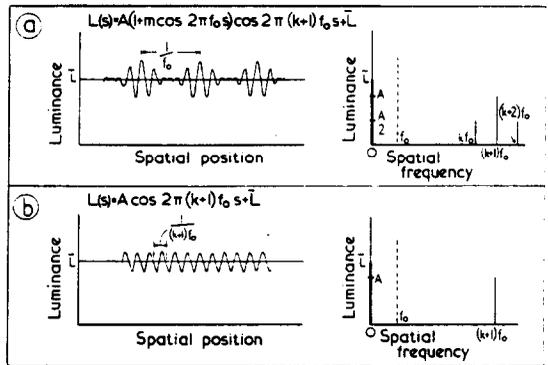


Fig. 3. (a) shows the distribution of luminance and the equivalent spectral representation of a contrast modulated grating; (b) shows the luminance distribution and equivalent spectral representation of the unmodulated grating the *O*s were required to distinguish from that represented in (a).

made by one *O* as a function of the percentage depth of modulation. Open symbols represent performance against a uniform background; closed symbols performance against the 1.9 c/deg masking grating with 17.4 per cent contrast. The *O*'s ability to detect sidebands or, alternatively, the ability to detect modulation in the contrast of the carrier, is markedly affected by the presence of the low-frequency masking grating. The *O* requires nearly four times more contrast to achieve the same percentage of correct responses when the low-frequency mask is present than when it is not. Similar results for the other *O* are shown in Fig. 5.

Discussion

The results of this experiment, together with those of the preliminary experiment show that a low-frequency grating can significantly influence the detectability of the components of a complex high-frequency grating. The interaction occurs even when the low-frequency grating has no measurable effect on that component of

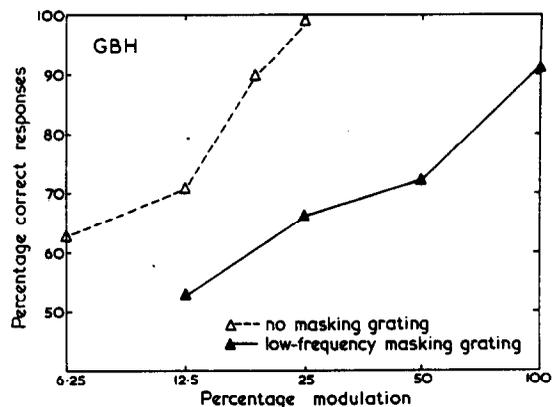


Fig. 4. This shows the percentage of correct detection of the sidebands as a function of the depth of modulation,  $m$ , expressed as a percentage. The open symbols represent the case in which no masking grating was used, the closed symbols the case with a 1.9 c/deg masking grating having 17.4 per cent contrast. Each data point is based on 100 observations by observer GBH.

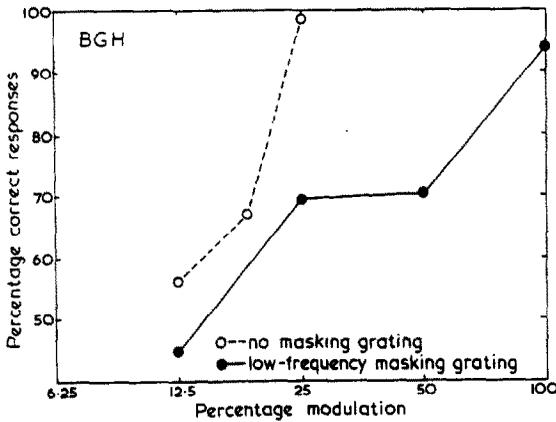


Fig. 5. This shows data for observer BGH similar to that shown in Fig. 4.

the high-frequency stimulus with the lowest spatial frequency. In the next experiment we explored the obverse effect, the effect of a high-frequency complex grating on the detectability of a low-frequency sinusoidal grating.

#### EXPERIMENT 2

##### Procedure

In this experiment, the signal to be detected was a 1.9 c/deg sinusoidal grating. The conditions of the previous two experiments were used again. Within each block of 50 trials we used one of three masking conditions; no masking grating; a three-component masking grating of the sort pictured in Fig. 3(a) with a carrier frequency of 9.5 c/deg, a modulation frequency of 1.9 c/deg, a carrier contrast of 27.1 per cent, and 100 per cent depth of modulation ( $m = 1.0$ ); or a 7.6 c/deg sinusoidal masking grating (equal in frequency to the lower sideband of the three-component grating) with 32 per cent contrast. The phase of the masking gratings was constant for any observation interval but varied from trial to trial. The signal phase was constant.

##### Results

Figure 6 shows the percentage correct detection of the 1.9 c/deg signal grating as a function of its contrast.

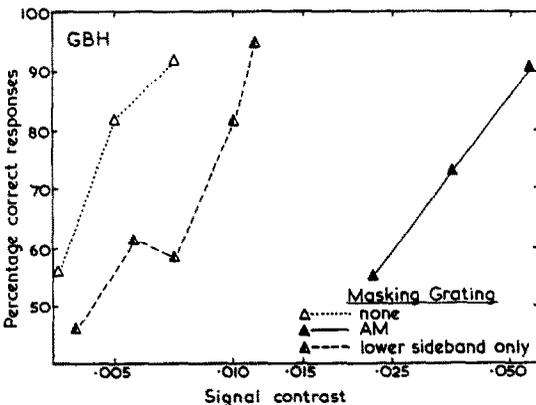


Fig. 6. This shows the percentage correct detection of a 1.9 c/deg sinusoidal grating as a function of the signal contrast. The performance for observer GBH is shown for three different maskings conditions: the open symbols show data with no masking grating present, the half-filled symbols the case with 7.6 c/deg masking grating, and the filled symbols are for the case in which a 100 per cent modulated carrier of 9.5 c/deg and 27.1 per cent contrast was present.

The open symbols represent data from the condition in which no masking grating was used; the half-filled symbols from the case in which a 7.6 c/deg sinusoidal masking grating (the lower sideband only) was used. The lower sideband used alone as a masker produces a masking effect. This result may be contrasted with that of the preliminary experiment (Figs. 1 and 2) in which no measurable masking effect was found when the spatial frequencies of the signal and masker were reversed. However, the masker contrast in this case is nearly twice that of the masker in experiment 1. Nevertheless, the asymmetry is not inconsistent with that found in other masking experiments (Stromeyer and Julesz, 1973; Henning and Hertz, 1974; Sansbury, 1974).

A much greater masking effect was produced by the contrast-modulated masker. The appropriate data are indicated by filled symbols. The  $O$  required 0.5 per cent contrast to achieve 75 per cent correct detection against a uniform background, 0.9 per cent against the 7.6 c/deg sinusoidal masking grating, but 3.8 per cent contrast against the contrast-modulated grating. Similar effects are shown in Fig. 7 for the other  $O$  for whom the masking effect of the modulated grating is not so large.

##### Discussion

The results of the two experiments together with those of the preliminary experiment show clearly that complex, high-frequency pattern and a low-frequency sinusoidal grating widely separated in spatial frequency can affect one another even though there is little interaction between sinusoidal gratings similarly spaced in frequency. The findings suggest that the detectability or discriminability of spatial patterns cannot be predicted from models assuming independent processing of stimulus components sufficiently widely spaced in spatial frequency. Before accepting this conclusion, however, alternative hypotheses in which systems of linear filters roughly tuned to different bands of spatial frequency might serve as an adequate model of at least one stage in pattern recognition (Pollen, Taylor and Lee, 1971; Ginsberg, 1974) should be considered. Harmonic distortion operating in conjunction with a set of bandpass filters provides a process potentially capable of producing our results.

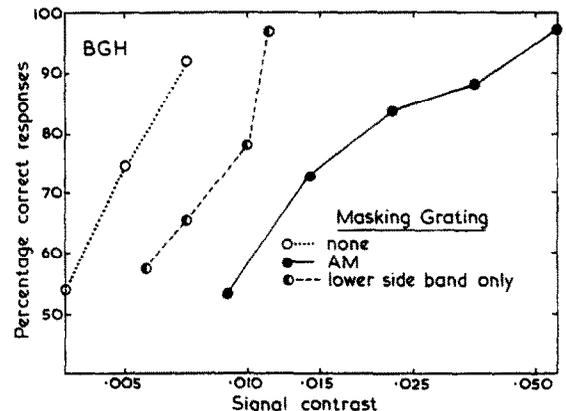


Fig. 7. This shows data for observer BGH from the conditions described in Fig. 6.

That the response of the visual system to patterns of light is a non-linear function of the luminance of the pattern has frequently been argued and will not be questioned here. Nor do we wish to consider the contention that, in principle, the non-linearity of the visual system precludes the effective use of models comprised of sets of crudely tuned linear analysing mechanisms (or channels); the success or failure of such models in predicting a wide range of results for the small signal case with constant adaptation level provides the appropriate source of information for their evaluation. Clearly most of what we look at in the real world is comprised of spatial transients with low contrast in any given band of spatial frequency. On the other hand, some, but not all, of our results might be interpreted as a demonstration of the effects of non-linearities in the visual system operating prior to a set of mechanisms tuned to narrow bands of spatial frequency. In fact, suitable assumptions about non-linearity will allow us to predict the results of experiments 1 and 2 fairly well; that is, the difficulty of detecting modulation in the contrast of a high frequency sinusoid in the presence of a low-frequency one, and of detecting the presence of a low-frequency sinusoid when masked by a contrast modulated high-frequency one. However, the assumption of a non-linearity severe enough to predict these results would require even a simple low-frequency sinusoid to produce a degree of masking at the spatial frequencies of its lower harmonics. This prediction is inconsistent with the results of our preliminary experiment. Thus, although our results are partially consistent with the occurrence of non-linearity and subsequent analysis in channels, such a model will not allow us to account for all our data. It is the assumption of non-linearity that we elaborate below.

In experiment 1, *Os* were required to detect a signal comprised of the two sidebands of a contrast modulated grating. When a masking grating of low spatial frequency was present, the task was harder—the sidebands required more contrast to be detected—than when no low-frequency masking grating was present. From Figs. 4 and 5, equation (2), and the knowledge that the carrier contrast was 6.3 per cent we can calculate the contrast in each sideband at the level corresponding to 75 per cent correct detection. With the low-frequency masking grating present, the levels were 1.6 and 1.9 per cent for *Os* GBH and BGH respectively. The calculated contrasts are not greatly above the level of 1.2 and 1.6 per cent (from Figs. 1 and 2) required by the *Os* to detect a sinusoidal grating at the spatial frequency of the lower sideband. We might conclude that, in the presence of a low-frequency masking grating, contrast modulation becomes apparent when the sidebands produced by the modulation are detected. This interpretation would not be inconsistent with the channel hypothesis. The *Os*, however, detected the contrast modulation at much lower levels of sideband contrast when there was no low-frequency masking grating present. Indeed, the contrasts of the sidebands corresponding to 75 per cent correct responses (0.004 and 0.006) were at least a factor of three lower than the level at which the lower sideband alone could be detected. (This finding is inconsistent with that reported by Bodis-Wollner, Diamond, Orlofsky, and Levinson, 1973.) It is possible that, with no low-frequency masking grating present, the *Os* detected

the presence of contrast modulation by detecting a low-frequency component not present in the stimulus but introduced by non-linearities in the visual system prior to the channels. The recent experiments of Burton (1973) indicate that this might well be the case and we wish to make a quantitative estimate of the non-linearity.

In order to explore further the hypothesis that visual non-linearities produce the interaction between high-frequency complex gratings and low-frequency sinusoidal gratings, two more experiments were performed.

### EXPERIMENT 3

#### Procedure

Experiment 3 differed from experiment 2 only in that the masking stimuli were different. In experiment 2 the masking stimulus was a contrast-modulated grating given by equation (2) with  $m$  (depth of modulation) equal to 1,  $\bar{L}$  (the mean luminance) equal to 1.55 ft-L, and  $K$  (the carrier contrast) equal to 6.3 per cent. The masking grating in experiment 3 was also a three-component grating with the same values of  $m$ ,  $\bar{L}$  and  $K$  but with the component at the frequency of the carrier,  $f_c$ , shifted by a quarter period; that is, the spatial frequency and amplitude of each component was the same as in experiment 2, but the relative phase of the three components was different. The luminance pattern of this masking grating was thus given by

$$L(s) = \bar{L} \{ K[(m/2) \sin 2\pi(f_c - f_m)s + \cos 2\pi f_c s + (m/2) \sin 2\pi(f_c + f_m)s] + 1 \}. \quad (3)$$

Figure 8(a) shows the contrast-modulated waveform of experiment 2; Fig. 8(b) that of experiment 3. This waveform is often called a quasi-frequency modulated waveform. It has the same autocorrelation function as the contrast-modulated grating of Fig. 8(a) but, unlike that grating, has a pronounced variation in its spatial frequency. It also has a slight amplitude modulation and can be written in the form

$$L(s) = \bar{L} \{ K(1 + m^2 \cos^2 2\pi f_m s)^{\frac{1}{2}} \cos[2\pi f_c s + \tan^{-1}(m \cos 2\pi f_m s)] + 1 \}, \quad (4)$$

where both the phase modulation and residual amplitude

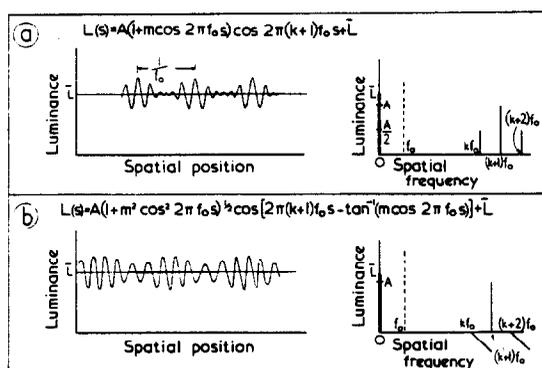


Fig. 8. (a) shows the distribution of luminance and equivalent spectral representation of the 100 per cent contrast modulated grating used as a masker in experiment 2. (b) shows the luminance distribution and equivalent spectral representation of the "quasi-frequency modulated" grating used as a masker in experiment 3. The amplitudes and spatial frequencies of the components of the gratings in (a) and (b) are identical: only the relative phase of the components is different. The two waveforms have the same autocorrelation function.

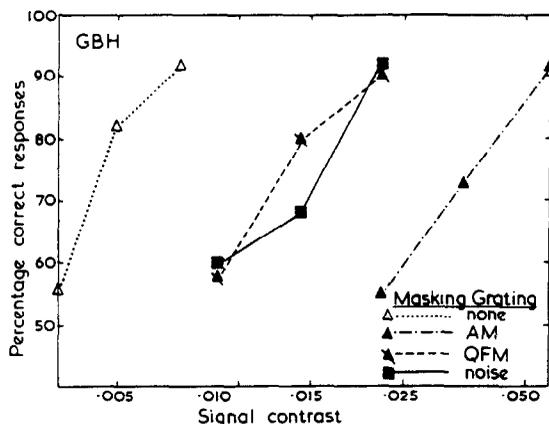


Fig. 9. This shows the percentage correct detection of a 1.9 c/deg sinusoidal grating as a function of the signal contrast. Data for experiment 2 with no masking grating (open symbols) and a 100 per cent contrast modulated masking grating are shown together with the data for the "quasi-frequency modulated" masker (symbols with a stroke through them) and noise band masker (square symbols).

modulation may be seen. The crucial feature of this waveform from our point of view is that the effective stimulus produced by the frequently assumed logarithmic non-linearity has almost no distortion product at the low spatial frequency of the modulation. We should expect, on the assumption of a logarithmic non-linearity, that this stimulus would have much less effect on the detectability of a low-frequency grating.

In addition to the quasi-frequency modulated grating, we also used a masking stimulus comprised of a large number of gratings of random amplitude and phase but all lying within the frequency band from  $(f_c - f_m)$  c/deg to  $(f_c + f_m)$  c/deg. This stimulus is a "noise" grating of uniform contrast power-density within that band. The noise grating had an average root-mean-square contrast equal to that of the contrast-modulated grating of equation (2). Under the logarithmic transformation, the noise stimulus would produce only negligible distortion products in the low-frequency region. A two-interval forced-choice detection task identical to that used in experiment 2 was used with the same two Os and the same equipment.

### Results

Figures 9 and 10 show the results of this experiment together with those of experiment 2 for comparison. In Fig. 9, the percentage of correct detection of a 1.9 c/deg sinusoidal grating is shown as a function of the signal contrast. Each data point is based on 100 observations from GBH. The results from experiment 2 in which no masking grating (open symbols) or a contrast modulated masking grating (closed triangles) were used are shown for comparison. It will be seen that neither the quasi-frequency modulated grating (closed triangles with a stroke through them) nor the band of visual noise has as large a masking effect as the contrast-modulated masker. Indeed, in the presence of the high-frequency noise, the O requires only 1.2 per cent contrast to detect the low-frequency sinusoidal signal 75 per cent of the time. This level is not greatly different from the 0.93 per cent contrast needed to detect the same grating in the presence of the lower sideband of the masking grating alone (Fig. 6). The results for the second O are somewhat different in that the quasi-frequency modulated grating produces almost no mask-

ing whatever. The noise-band-grating on the other hand produces slightly more masking than the high-frequency sinusoidal grating (lower sideband alone) of experiment 2.

### Discussion

It is possible to interpret these data as support for a visual non-linearity's determining the effects of a complex grating; where there would be only a small low-frequency distortion product, as in the noise and quasi-frequency modulated masking case, there is little masking; where there would be a large distortion product, the contrast modulated case, there is a great deal of masking.

## EXPERIMENT 4

### Procedure

The fact that high-frequency complex gratings likely to produce minimal distortion products at the spatial frequency of the signal have considerably less masking effect than complex gratings that could have a large distortion product at the signal frequency led us to a final experiment in which the phases of both the masking and signal grating were fixed. The masking grating was the low-frequency sinusoid of experiment 1. However, instead of varying in phase from observation interval to observation interval, the grating was fixed in one of three different positions for each block of trials. The observers were required to detect the addition of the sidebands to the carrier frequency as in experiment 1.

In Appendix I, it will be seen that the distortion product produced by the hypothetical logarithmic visual non-linearity is in negative cosine phase [equation (7) of the Appendix]. That is, the black bar of the distortion product should occur directly under the high contrast part of the contrast modulated grating.

In experiment 4, the task faced by an observer attempting to detect a signal by the presence of an hypothetical distortion product produced by the signal depends on the relative phase of the distortion product and the low-frequency sinusoidal masker; if the two components are in phase, the O's task is one of increment detection. The contrasts of the masker and the distortion product (signal) add, and the effect of the mask will depend on the level of the distortion product in the way described by Campbell and Kulikowski (1966) in their study of the interaction of two sinusoidal gratings of the same orientation and frequency. If the masker and the distortion product are 180° out of phase, the O will be required to detect a decrement in the contrast of

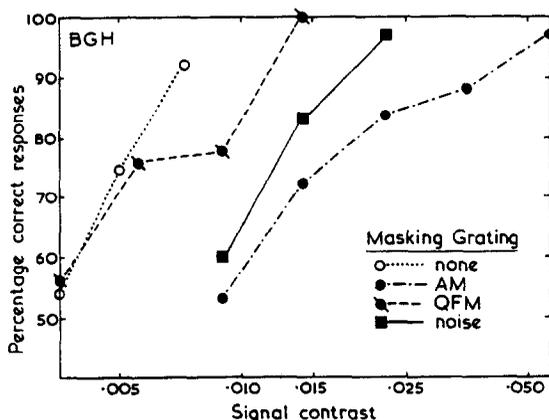


Fig. 10. This shows data for observer BGH in the conditions described in Fig. 9.

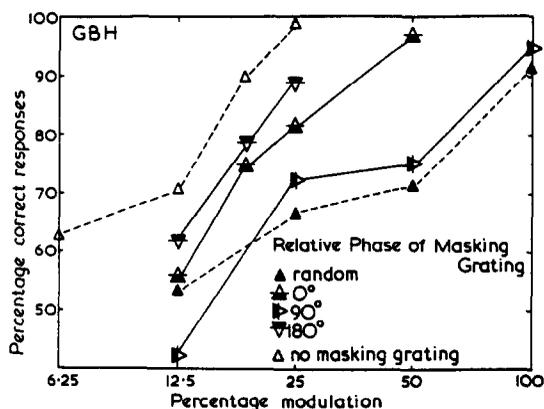


Fig. 11. This shows the percentage of correct detection of the addition of sidebands as a function of the depth of modulation. Performance was measured with a low-frequency sinusoidal masking grating of fixed phase in both observation intervals. Three different phase relations are shown together with the results from the cases in which no masking grating and one of random phase were used.

the component at the masker frequency if the contrast of the distortion product is less than that of the masker. The size of the decrement will depend on the relative amplitude of the mask and the distortion product and will become an increment (with  $180^\circ$  phase shift) when the distortion product contrast exceeds twice that of the masker. If the masker is shifted by either  $90^\circ$  or  $270^\circ$  of phase relative to the distortion product, the result of adding the distortion product and the mask will be a component with contrast is equal to the square root of the sum of the squares of the masker and distortion product contrasts. The resultant will be shifted in phase by an amount dependent on the relative amplitudes of the two components.

Experiment 4 was identical to experiment 1, save that the phase of the low-frequency masking grating was fixed for any given block of 50 trials. Three different phase conditions were used: the sinusoidal masking grating and the modulation factor  $[\cos 2\pi f_m t]$  in equation (1)] were: (a) in phase, that is, with the bright bar of the masker added to the high contrast region of the contrast modulated signal grating; (b)  $180^\circ$  out of phase, that is, with the dark bar of masker added to the high contrast region of the signal grating; or (c)  $90^\circ$  advanced, that is, with the dark bar of the masking grating added to the region of the signal grating where the contrast was decreasing to the right and half way between the maximum and the minimum contrast of the modulated signal grating.

### Results

Figures 11 and 12 show the percentage of correct detection of the sidebands as a function of the depth of contrast modulation. (It should be remembered that depth of modulation is proportional to sideband contrast.) Three different conditions of masker phase are shown together with the random phase condition and the condition of no masking grating from experiment 1. The data for each  $O$  are shown separately in the two figures and each data point is based on a minimum of 100 observations.

The results for the fixed phase cases are clear; there is little difference between the masking effects of the in-phase and  $180^\circ$  out-of-phase conditions—and those effects are small. The  $90^\circ$  phase condition produces the most masking—very nearly the same amount as that produced by the random-phase masker of experiment 1.

### Discussion

The results of experiment 4, like those of our other experiments, have features that are consistent with the hypothesis of a logarithmic non-linearity, and features that are not. The difference in the detectability of modulation based on a distortion product assumed to be in-phase with the low-frequency masking grating and that assumed to be  $90^\circ$  out of phase is consistent on the following argument:

Suppose that the modulation is detected by comparing the effective amplitude of the component of the stimulus at  $1.9$  c/deg in the two observation intervals. When no signal is present, this component will derive from the masker alone. When the sidebands are added, a distortion product at  $1.9$  c/deg results and, in the case in which the masker and the distortion product are in the same phase, will add its contrast to that produced by the masking grating to produce an increment in the effective contrast. The detectability of that increment will be consistent with Weber's Law—the detectable increment will be proportional to the background contrast—and we can determine that the ratio of the contrast at  $1.9$  c/deg in the two intervals need be about 1.15 from Campbell and Kulikowski (1966). The increment produced by adding the sidebands in the " $90^\circ$ " condition of phase is less than the increment produced in the "in-phase" condition. If the contrast produced by the signal at the spatial frequency of the masker is  $c_1$ , and that produced by the masker  $c_2$  then the resulting contrast will be  $\sqrt{c_1^2 + c_2^2}$ . To achieve a detectable increment in the " $90^\circ$ " case, the effective contrast of the distortion product,  $c_1$ , will have to be about four times as great as in the "in-phase" case. Equation (9A) indicates that to achieve such an increase in the effective contrast of the distortion product under a logarithmic transformation the sideband contrast must also be increased by a factor of 4. Our observers required increases of almost that much between the stimulus contrasts leading to 75 per cent correct detection in the two conditions. (That ratio is not obtained at other performance levels, 55 per cent correct or, say, 95 per cent. The slopes of the functions relating performance to sideband contrast change with the phase condition but cautious readers will already be aware of the difficulties in drawing inferences about underlying mechanisms on the basis of one contour through the stimulus space of a few experiments.)

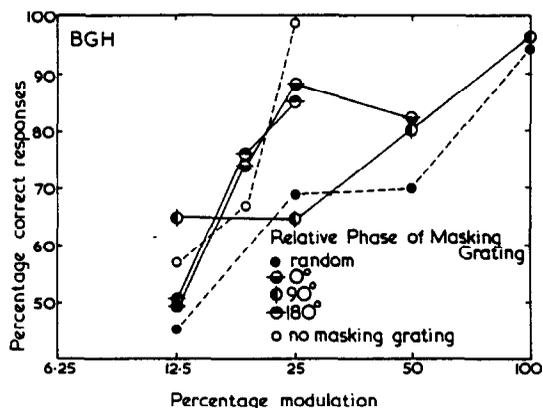


Fig. 12. This shows results for observer BGH in the conditions outlined in Fig. 11.

The fact that the "in-phase" and "180° out-of-phase" conditions produce almost no masking is not necessarily inconsistent with the hypothesis of non-linearity but is not explained by that assumption. There are major differences in the *O*'s approach to fixed- and random-phase masking gratings. With fixed-phase gratings the *O*s were able to note and learn spatial properties of the masker alone and signal plus masker gratings. The location of a narrow white bar relative to the minimum luminance of the low-frequency masking grating might for example, serve to distinguish signal from noise in one phase condition and the *O*s reported using such cues. Reliable cues of the same sort are not available in the "random-phase" conditions. The *O*s were thus using location or phase cues in the "fixed-phase" but not in the "random-phase" case, but there is no obvious reason for such cues to be so much less helpful in the 90° fixed-phase case than they are in the 0 or 180° fixed-phase cases. An explanation does not readily come from the assumption of a visual non-linearity.

#### DISCUSSION

It is clear from our results that the detectability and discriminability of spatial patterns cannot be predicted from a knowledge of their Fourier spectra and of the response of the visual system to each component of the spectrum, even when those components have widely differing spatial frequencies. This is disappointing; a crude spatial-frequency analysis in broadly tuned linear and independent channels would have provided an attractive and simple model of human pattern recognition.

In order to preserve such a model and still account for our data, we explored the possibility that a non-linear transformation of luminance operating prior to a linear spatial frequency analysing mechanism could predict our results.

Figures 7 and 8 show, as open symbols, the contrast needed to detect a sinusoidal grating of 1.9 c/deg—the modulation frequency of our complex waveform. If we knew the form of the visual non-linearity, we could calculate the amount of distortion needed to produce from our complex waveform a component of 1.9 c/deg equivalent in contrast and spatial frequency to that produced by the sinusoidal grating of that frequency.

There are a number of ways we could set about determining the form of the non-linearity. (A typical set of calculations may be found in the Appendix.) One general technique might be to assume an arbitrary polynomial non-linearity such that the effective stimulus,  $y(s)$ , produced by the stimulus,  $x(s)$ , is given by,

$$y(s) = Ax(s) + Bx^2(s) + Cx^3(s) + \dots, \quad (5)$$

where  $A, B, C, \dots$  are arbitrary constants to be determined from our data.

An alternative approach would be to accept the form of non-linearity suggested from other experiments. Davidson (1966), Cornsweet (1970), Burton (1973), and Maudarbocos and Ruddock (1973) have all argued in favour of a logarithmic non-linearity in which the effective stimulus,  $y(s)$ , produced by a stimulus,  $x(s)$ , is given by

$$y(s) = \log[x(s)]. \quad (6)$$

Where our results are consistent with the hypothesis of

a non-linear visual system, they are also consistent with that non-linearity's being logarithmic: that is, the coefficients of the best fitting quadratic non-linearity have the values one would expect to find if the non-linearity were logarithmic. It seemed reasonable, then, particularly in view of the limited amount of data we have, to assume the hypothetical non-linearity to be logarithmic and the effective stimulus given by equation (6).

In our experiments, the stimulus,  $x(s)$ , is of the form

$$x(s) = \bar{L}[1 + f(s)] \quad -1 \leq f(s) \leq 1 \quad (7)$$

where  $\bar{L}$ ,  $f(s)$  is the spatially varying component and  $\bar{L}$  the mean luminance.

The effective stimulus in response to  $x(s)$  is thus  $y(s)$  given by

$$y(s) = \log \bar{L} + \log[1 + f(s)] \quad -1 \leq f(s) \leq 1. \quad (8)$$

The term  $[1 + f(s)]$  may be thought of as a function representing the normalized luminance pattern; the luminance at each point relative to the mean luminance. Equation (8) may be expanded in a series such that

$$y(s) = \log \bar{L} + \{f(s) - [f^2(s)/2] + [f^3(s)/3] - \dots\} \quad -1 < f(s) \leq 1. \quad (9)$$

(The additional constraint on  $f(s)$  is met in all our stimulus conditions.) If  $f(s)$  is a spatially periodic function with period  $S_0$  degrees, as it is assumed to be in all our experiments, then  $y(s)$ , the effective stimulus, will be comprised of a number of sinusoidal components with amplitudes that depend on the stimulus,  $x(s)$ , but with spatial frequencies that are always integer multiples of the fundamental spatial frequency,  $1/S_0$  c/deg. There will also be an effective mean luminance term,  $\log \bar{L}$ , and a number of mean luminance terms contributed by the non-linear operations on  $f(s)$ -squaring, cubing, and so on.

A question arises as to which aspects of the effective stimulus—which components of  $y(s)$ —we are to take as determining the *O*'s ability to detect, and discriminate among, different stimuli. We might take the effective amplitude of a component in  $y(s)$ , we might take the ratio of the amplitude of that component to the effective mean luminance, or we might even take the contributions of the non-linearities to the effective mean luminance.

The last possibility we rejected because the effective mean luminance contributions of the non-linearities arising from the addition of our signals were in all cases far too small to be detected—that is, a great deal smaller than the Weber fraction for the detection of changes in mean luminance would suggest to be appropriate. The amplitude of the distortion product produced from the contrast-modulated grating by a logarithmic non-linearity is much smaller than that required to detect a grating at the spatial frequency of the hypothetical distortion product and so we chose to use the second possibility because it provides one free parameter, the unit in which we represent luminance. The extra degree of freedom can be determined from our data and gives the hypothesis that our results are determined by visual non-linearities a greater chance of correctly predicting those results.

By appropriately selecting the unit of mean luminance, we can force the ratio of the amplitude of the distortion product to the effective mean luminance

to be equal to the ratio of the amplitude of an equally detectable sinusoidal grating of the same spatial frequency to the effective mean luminance. We were able to test the hypothesis of non-linearity by using the estimated value of  $\bar{L}$  to estimate the contrast produced at a spatial frequency of  $2f_0$  by the non-linear distortion of a masking stimulus of frequency  $f_0$  and comparing that value with the value produced by the signal of frequency  $2f_0$ . We used the data from our preliminary experiment with  $2f_0$  equal to 7.6 c/deg. The masking stimulus would, we found, produce more effect at the spatial frequency of the signal than three times that produced by a signal corresponding to 75 per cent correct responses. It seems unlikely that such a component would produce no measurable effect on the detectability of the signal and we concluded that the results of our preliminary experiment are inconsistent with the existence of a non-linearity sufficient to produce the results of experiment 1.

We can force some of our data to be consistent with the common assumption of a logarithmic non-linearity but that assumption predicts large interactions between sinusoidal gratings that we fail to find. We are unable to make a combination of processes in which a non-linear mechanism precedes a linear frequency analysing one predict even our limited set of data, unless we accept the remote, but testable, proposition that the non-linearity is confined to the region of high spatial frequencies.

Our stimuli, with the exception of the noise masker, were all harmonically related. We cannot say whether the interactions we find are influenced by this fact. A brief attempt was made to determine if the masking effect of a complex grating of high spatial frequency was maximal at the period of that grating, that is, at the frequency corresponding to the period of the complex grating where no spatial component was present. Our result indicated that there was no maximum at the period of the masking grating but because of limitations on the range of stimuli we could generate we are reluctant to proceed on the basis of this result without doing more experiments.

On the other hand it is quite clear from a similarly brief experiment that high-frequency sideband signals are effectively masked by bands of low-frequency noise; the low-frequency mask need not be sinusoidal.

We are unable, yet, to determine whether the interactions we obtain can be obtained with gratings in different eyes. It would also be interesting, for example, to know if changes in the apparent depth of a binocular grating composed of a high-frequency grating in one eye and a sinusoidal grating of the same period on the other arise from changes in the phase of the gratings. We should also like to know if the detectability of contrast increments or spatial frequency shifts in high-spatial frequency carriers are generally masked by low-frequency gratings. Or whether these changes must have a low-frequency spatial variation in order to be masked.

Finally, it is just conceivable that our results indicate something about the early organization of the mechanisms underlying the spatial frequency tuning found in some cortical neurones by Cooper and Robson (1968). It is clear that the receptive field organization of a particular cell determines that the cell will have a particular spatial-frequency response and we might think of a receptive field at any level as arising

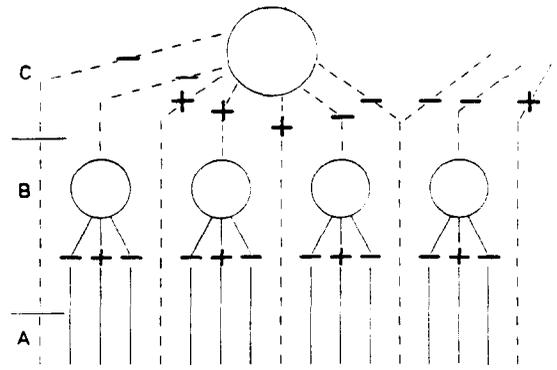


Fig. 13. This shows one hypothetical arrangement of mechanisms that would account for our data. Elements at level A are sensitive to luminance. Elements at level B are sensitive to sinusoidal gratings of high but not low spatial frequency. Elements at level C are sensitive to sinusoidal gratings of low but not high spatial frequency and also to low frequency variations in the contrast of a grating of high spatial frequency.

from the interactions among a number of elements responding to some function of the stimulus luminance. Different patterns of spatial organization would give rise to different frequency-response characteristics and we might build a system of tuned and independent "channels" with the properties that the channels were each selectively sensitive to different bands of spatial frequencies. If, however, the channels tuned to high spatial frequencies were to act as elements, together with luminance sensitive elements, for channels sensitive to low-frequency bands then all our results might be predicted. Figure 13 shows a functional scheme consistent with our data. The scheme should be considered merely as one possible functional equivalent of part of the visual system. Luminance sensitive elements at level A affect elements at two subsequent levels, B and C. Plus signs indicate an excitatory effect and minus signs an inhibitory one. At level B the connections are such that a given element will respond to a sinusoidal grating of high but not low spatial frequency. At level C, on the other hand, the elements will respond to a sinusoidal grating of low but not high spatial frequency. However, if a grating of high spatial frequency varying in contrast is presented, then those elements at level B in the high contrast region of the grating would respond strongly and those in the low contrast regions weakly. The spatial variation in the response of the high-frequency sensitive elements at level B would lead to a response to the contrast modulated grating by the elements at level C. This linear system would exhibit little interaction between sinusoidal gratings of widely differing spatial frequency but would show interaction between low-frequency sinusoidal gratings and contrast-modulated high-frequency ones in a fashion consistent with our results. This speculation implies a particular form of neurophysiological organization and might profitably be ignored until rather more physiological evidence is available.

#### SUMMARY

(1) Our experiments demonstrate that a low-frequency sinusoidal grating and a complex of three high-frequency sinusoidal gratings interact and that

the two types of grating interact even when there is no measurable interaction between the low-frequency grating and the lowest frequency component of the high-frequency complex.

(2) This finding is inconsistent with the hypothesis that the visual system processes spatial patterns in independent linear mechanisms tuned to limited bands of spatial frequency.

(3) Our results do not support the hypothesis that the visual system responds to the pattern produced by a logarithmic transformation of luminance followed by a linear analysis in independent bands of spatial frequency: such a transformation is consistent with some of our data, but a logarithmic non-linearity should be, and is not, apparent in the interaction of a high-contrast low-frequency masker and a signal at the second harmonic of the masker.

(4) The data are consistent with a non-linear distortion of signals of high, but not low, spatial frequency, but this unlikely hypothesis has yet to be tested in detail.

(5) The data could also be predicted by a visual system in which an array of elements was arranged to respond to luminance variation within a number of bands of spatial frequency provided that the channels responding to low-frequency variations also contained, as elements, channels responsive to high spatial frequencies.

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#### APPENDIX 1

It is sometimes assumed that the effective luminance in the visual system is a logarithmic transformation of the stimulus luminance, so that a stimulus,  $x(s)$ , produces an effective stimulus,  $y(s)$ , where

$$y(s) = \log[x(s)]. \quad (1A)$$

We shall begin with this assumption.

The stimulus patterns used in our experiments may all be written in the form

$$x(s) = \bar{L}[1 + f(s)], \quad -1 \leq f(s) \leq 1 \quad (2A)$$

where  $f(s)$  is a term of either one or three sinusoidal components having zero mean and whose total amplitude never exceeds unity. We may therefore expand equation (1A) in a power series of the form

$$\log[x(s)] = \log \bar{L} + f(s) - f^2(s)/2 + f^3(s)/3 - \dots \quad (3A)$$

For a sinusoidal grating,  $f(s)$  is given by

$$f(s) = K \sin(2\pi f_0 s), \quad 0 \leq K \leq 1 \quad (4A)$$

where  $K$  is the contrast of the grating and  $f_0$  its spatial frequency. Substituting this expression into equation (3A) we find the effective stimulus in response to a sinusoidal grating to be given by

$$y(s) = \log \bar{L} + K \sin 2\pi f_0 s - (K \sin 2\pi f_0 s)^2/2 + (K \sin 2\pi f_0 s)^3/3 - \dots \quad (5A)$$

For the stimuli we consider first, contributions from terms of power greater than two will be about 100th the size of the linear term and will be neglected, that is,  $y(s)$  will be approximated by the first three terms of equation (5A). Thus,  $y(s)$  for a sinusoidal stimulus is given approximately by

$$y(s) \simeq \log \bar{L} - K^2/4 + K \sin 2\pi f_0 s + (K^2/4) \cos 2\pi(2f_0)s. \quad (6A)$$

When there is no grating present, only the uniform mean luminance of the display, the value of  $K$  in equation (6A) is zero. When a sinusoidal grating having the same mean luminance is present, equation (6A) shows that, in addition to a component at the spatial frequency of the grating, there will be a decrement of  $K^2/4$  in the effective mean luminance

and a sinusoidal term of the amplitude  $K^2/4$  varying at twice the spatial frequency of the stimulus. In this general formulation the importance of the various terms relative to the mean luminance depends, unfortunately, on the choice of the units in which the luminance is measured. Further, the observers may use any of three terms (or indeed any combination of the three) to detect the signal.

The transformation of equation (3A) applied to a contrast modulated waveform yields a much more complex effective stimulus in which

$$y(s) \approx \log \bar{L} - (K^2/4) - (K^2 m^2/8) - (K^2 m/2) \cos 2\pi f_0 s \\ - (K^2 m^2/8) \cos 2\pi(2f_0)s, \quad (7A)$$

plus additional terms at spatial frequencies of 4, 5, 6, 8, 9, 10, 11 and 12 times the modulation frequency  $f_0$ . We know from experiment 1 that the components at 4 and 6 times the modulation frequency—the sidebands—will not themselves be detectable at the unmasked threshold of modulation where they have amplitude  $\bar{L}Km/2$ . The only high-frequency components with greater amplitude are at 5 and 10 times the fundamental frequency. These are the carrier and its second harmonic; we should not expect combinations of the higher-frequency components to determine the detectability in any simple fashion even though many of them fall within an octave of one another and have the same initial phase. If this were to be the case, and we know multiple component signals in which the components are within an octave or so of one another may be more detectable than we might predict from the detectability of each component alone (Graham and Nachmias, 1971) then it would be difficult to explain why the low frequency sinusoid has any masking effect whatever. We neglect, then, the higher frequency components and assume that, in the absence of low-frequency masking grating, the Observers detect the addition of sidebands in experiment 1 by detecting the decrement ( $K^2 m^2/8$ ), the component at  $f_0$ , or the component at  $2f_0$ .

Rather than inflict on the reader the tedium of our checking each of the several hypotheses, we show the detailed calculations for one case and simply state our other results. The case we treat in detail is the assumption that the observers detect modulation on the basis of their detection of that component of the effective stimulus at spatial frequency  $f_0$  and that it is the ratio of the effective amplitude of the component relative to the effective mean luminance that determines its detectability. If this were the case then that ratio in the effective stimulus resulting from a sinusoidal grating of frequency  $f_0$  should be equal to the effective contrast produced at that frequency by the non-linear transformation of the contrast modulated grating. The ratio,  $c_m$  for a sinusoidal grating of spatial frequency  $f_0$  determined from equation (6A) is given by

$$c_s = K_1 / (\log \bar{L} - K_1^2/4), \quad (8A)$$

where  $K_1$  is the contrast of the sinusoidal grating. The ratio,  $c_m$  at frequency  $f_0$  for the contrast-modulated grating may be determined from equation (7A) and is given by

$$c_m = (K_2^2 m/2) / [\log \bar{L} - (K_2^2/4) - (K_2^2 m^2/8)], \quad (9A)$$

where  $K_2$  is the contrast of the carrier and  $m$  is the depth of modulation. By hypothesis  $c_s$  is equal to  $c_m$  and we may solve that equation for  $\bar{L}$  by substituting the values of  $K_1$ ,  $K_2$ , and  $m$  corresponding to 75 per cent correct detection in the two cases. We call the estimate  $\hat{\bar{L}}$ . (The value obtained will be subject to errors in the estimation of  $K_1$  and  $m$  and we estimate the error in  $\hat{\bar{L}}$  by finding solutions for values of  $K_1$ ,  $K_2$  and  $m$  ranging within a standard deviation of the values corresponding to 75 per cent correct responses.) For GBH,  $K_1$  is  $0.0045 \pm 0.0003$ ,  $K_2$  is  $0.063$  and  $m$  is  $0.122 \pm 0.0005$ . With these values  $\hat{\bar{L}}$  is  $1.0024 \pm 0.00007$  for observer GBH. For observer BGH,  $\hat{\bar{L}}$  is  $1.0026 \pm 0.00003$ . (It is clear from values of  $K_1$ ,  $K_2$  and  $m$  that the effective amplitude is not the appropriate variable, that is,  $K_1$  does not equal  $K_2^2 m/4$ .)

The values of  $\hat{\bar{L}}$  are consistent, by definition, with our data on the following assumptions made in determining the transformation of our signals:

(1) The effective stimulus,  $y(s)$ , is the logarithm of the stimulus luminance,  $x(s)$ , and is well approximated by a quadratic equation.

(2) The ratio of the amplitude of a component in  $y(s)$  to the effective mean luminance is the appropriate characteristic to determine the detectability of gratings.

(3) The addition of sidebands in experiment 1 is detected by a distortion product at the spatial frequency of the contrast modulation when that distortion product has an effective contrast equal to that produced at that frequency by a sinusoidal grating of that frequency.

Under these assumptions and using the values of  $\hat{\bar{L}}$ , we can use equation (6A) to predict the masking effect of a sinusoidal grating on signals of twice the spatial frequency of the masking grating.

Let us now consider alternative hypotheses and show that the decrement in the effective mean luminance produced by adding spatially varying components cannot be used by the Os to detect the presence of a grating. We know from Weber's Law and the standard psychophysical measures what change in the mean luminance of a stimulus leads to detection of difference in luminance level. That is, if  $\bar{L}$  is the mean luminance then a "just-noticeable" increment  $\Delta L$ , will arise if

$$\bar{L} + \Delta L = 1.03\bar{L}. \quad (10A)$$

In terms of the effective stimulus resulting from our hypothetical non-linearity we find that

$$\log(\bar{L} + \Delta L) = 0.3 + \log \bar{L}. \quad (11A)$$

Using the values of  $K_1$  corresponding to 75 per cent correct responses from equation (6A) and the preliminary experiment, we find

$$\log(\bar{L} + L) = K_1^2/4 + \log \bar{L} = 0.00005 + \log \bar{L} \quad (12A)$$

and the effective decrement is much smaller than that required in detecting a luminance change. The decrement produced by adding a contrast-modulated grating is similarly too small to be detected.