# Colorimetry

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**Abstract**
Colorimetry is concerned with the measurement and numerical specification of the color of visual stimuli. Color depends on the underlying biology of the human visual system, and in the first instance on the spectral properties of the three types of light-sensitive cone photoreceptors in the eye. Color and color matches are trichromatic, meaning they can be specified by three variables that are linearly related to the outputs of the three cone types. The primary aim of colorimetry is to be able to specify those variables for lights of any spectral composition.

**Keywords**
color matching; color matching functions; cone fundamentals; colorimetry; photometry; trichromacy; univariance; cone spectral sensitivities; chromaticity diagrams; color, CIE.

## 1 Introduction

Colorimetry is a branch of color science pertaining to the measurement and numerical specification of the *color* of visual stimuli. Since perceived color is a property of the human eye and brain, and not a property of physics [1], colorimetry is inextricably linked to the underlying biology of vision (see also PHYSIOLOGICAL OPTICS).

Color and color perception are limited at the first stage of vision by the spectral properties of the layer of light-sensitive photoreceptors that carpet the rear surface of the eye (upon which an inverted image of the world is projected by the eye's optics). These *photoreceptors* transduce arriving photons to produce the patterns of electrical signals that eventually lead to perception. Daytime (photopic) color vision depends mainly upon the three classes of cone photoreceptors, each with different spectral sensitivity, which are referred to as long-, middle- and

short-wavelength-sensitive (L, M, and S), according to the part of the visible spectrum to which they are most sensitive (see Fig. 5). Nighttime (scotopic) vision, by contrast, depends on a single class of photoreceptor, the rod.

## 1.1 Trichromacy and Univariance

The range of colors that can be produced by the additive combination of just *three* lights is simulated in Fig. 1. Overlapping red, green, and blue lights produce regions that appear cyan, purple, yellow, and white. Other intermediate colors can be produced by varying the relative intensities of the three lights.

A fundamental property of normal human vision is that it is *trichromatic*: observers can match a test light of any spectral composition to a suitably adjusted mixture of just three other lights (with the proviso that one of the three primaries invariably has to be added to the first light in order to complete the match – see Sect. 1.2.1, and Eq. (1)). As a result, colors can be defined by just three variables: the intensities of the three primary lights with which they match (the three "Tristimulus" values).

Additive color mixing

**Fig. 1** Additive color mixing. Simulated overlap of projected red, green and blue lights. The additive combination of red and green is seen as yellow, red and blue as purple, green and blue as cyan, and red, green and blue as white

Trichromacy arises because there are only three classes of cone photoreceptor in the human eye, each of which responds univariantly to light in a way that depends on the rate of photon absorption [2, 3]. Although the *probability* that a photon is absorbed by a given photoreceptor varies by many orders of magnitude with wavelength, its effect, once absorbed, is *independent* of wavelength. An absorbed long-wavelength photon has the same effect as a short-wavelength one. Consequently, a photoreceptor is like a sophisticated photon counter, the output of which varies according to the rate of absorbed photons. Since a change in rate could result from a change in the wavelengths of the photons, from a change in the number of photons, or from both, individual photoreceptors are effectively color-blind. Color is encoded by the *relative* outputs of the three, individually color-blind cone types. It is therefore trivariant or trichromatic.

For background reading on color vision, see, for example, Kaiser & Boynton (1996). See also the COLOR VISION article.

## 1.2 Color Matching

Trichromacy means that the color-matching behavior of an individual can be straightforwardly assessed by determining the intensities of three independent fixed-wavelength "primary" lights (independent in the sense that no two primaries will match the third) required to match

a series of monochromatic spectral lights spanning the visible spectrum. Two principal methods have been used to determine color matches: the maximum saturation method and Maxwell's method. Most colorimetry is based on the former, though it is arguably the inferior method.

### 1.2.1 Maximum Saturation Method

The most commonly used matching method is the maximum saturation method, which was used by Wright [4] and Guild [5] to obtain matches that were subsequently used to generate the CIE 1931 Color Matching Functions (CMFs) (see Sect. 5.1). As illustrated in Fig. 2(a), the observer is presented with a half field illuminated by a "test" light of variable wavelength, λ, and a second half field illuminated by a mixture of red (**R**), green (**G**) and blue (**B**) primary lights (note that bold capitals denote primary lights). At each λ, the observer adjusts the intensities of the three primary lights, so that the test field is perfectly matched by

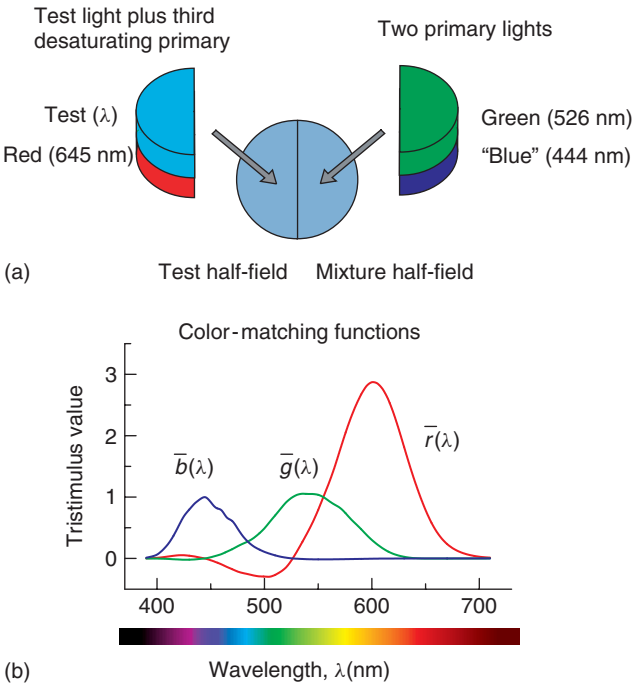


**Fig. 2** Maximum saturation method of color matching. A monochromatic test field of wavelength, λ, can be matched by a mixture of red (645 nm), green (526 nm) and blue (444 nm) primary lights, one of which must be added to the test field to complete the match. The amounts of each of the three primaries required to match monochromatic lights spanning the visible spectrum are known as the red, $\bar{r}(\lambda)$, green, $\bar{g}(\lambda)$, and blue, $\bar{b}(\lambda)$ CMFs (red, green, and blue lines respectively) shown in the lower panel. The data are from Stiles & Burch [6]. A negative sign means that that primary must be added to the target to complete the match

the mixture of primary lights. Figure 2(b) shows the mean $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ CMFs obtained by Stiles & Burch [6] for primary lights of 645, 526, and 444 nm. Notice that except at the primary wavelengths one of the CMFs is invariably negative. There is no "negative light"; rather, these negative values indicate that the primary in question has been added to the test light in order to make a match. Real primaries always give rise to negative values because lights cannot uniquely stimulate single cone photoreceptors, since their sensitivities overlap throughout the visible spectrum (see Fig. 5).

The general form of the maximum saturation match between $\mathbf{E}_\lambda$, a monochromatic constituent of the equal unit energy stimulus, **E**, of wavelength $\lambda$, and the three primaries lights ($\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$) is

$$\mathbf{E}_\lambda + \bar{a}_1(\lambda)\mathbf{A}_1 = \bar{a}_2(\lambda)\mathbf{A}_2 + \bar{a}_3(\lambda)\mathbf{A}_3, \quad (1)$$

where $\bar{a}_1(\lambda)$, $\bar{a}_2(\lambda)$, and $\bar{a}_3(\lambda)$ are the three CMFs set by the observer. For the special case of the unit energy stimulus, the *tristimulus values*, $A_1$, $A_2$, and $A_3$ (note the uppercase, italic convention), are simply the three CMF values. Otherwise, they are the CMF values scaled by the radiant power of the monochromatic test stimulus. Although CMFs are usually defined for a stimulus, **E**, that has equal unit energy throughout the spectrum, in practice, the spectral power of the test light used in most matching experiments is varied with wavelength. CMFs and the spectral power distributions of lights are nearly always discrete functions of wavelength, typically defined in steps of 1, 5, or 10 nm.

### 1.2.2 Maxwell's Matching Method

The first careful, quantitative measurements of color matching and trichromacy were made by Maxwell [7]. In Maxwell's method, which is illustrated in Fig. 3, the matched fields always appear white, so that at the match point, the eye is always in the same state of adaptation whatever the test wavelength (in contrast to the maximum saturation method in which the chromaticity of the match varies with wavelength). In a matching experiment, the subject is first presented with a white standard half field, and is asked to match it with the three primary lights. The test light

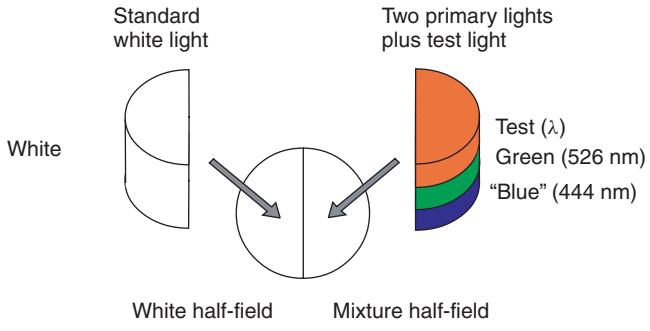


**Fig. 3** Maxwell's method of color matching. A monochromatic test field of wavelength, $\lambda$, replaces the primary light to which it is most similar, and a match is made to the white standard by adjusting the intensities of the two remaining primaries and the test field

then replaces the primary light to which it is most similar and the match is repeated.

### 1.2.3 Linearity and Additivity of Color Matches

Color-matching data, obtained for spectral lights, are useful *in general* only if they can be used to predict matches for nonspectral lights of any arbitrary spectral power distribution, and by extension for other triplets of primary lights. Color matches, in other words, must be linear and additive. If not, a new match would have to be established for each new matching condition. Grassmann's laws embody tests of linearity and additivity. They are as follows [8, 9]:

1. *Symmetry*: If light **A** matches light **B**, then **B** matches **A**.
2. *Transitivity*: If light **A** matches light **B** and **B** matches light **C**, then **A** matches **C**.
3. *Proportionality*: If light **A** matches light **B**, then $n$**A** matches $n$**B** (where $n$ is a constant of proportionality).
4. *Additivity*: If **A** matches **B** and **C** matches **D**, then the combination of **A** and **C** matches the combination of **B** and **D** (and similarly the combination of **B** and **C** matches **A** and **D**).

To a first approximation, color matching is linear and additive [9, 10].

### 1.2.4 Complex Stimuli

Given additivity, the tristimulus values, $A_1$, $A_2$, and $A_3$ for an arbitrarily complex spectral radiant power distribution, $P(\lambda)$ can be obtained from the $\bar{a}_1(\lambda)$, $\bar{a}_2(\lambda)$, and $\bar{a}_3(\lambda)$ CMFs, thus

$$A_1 = \int P(\lambda)\bar{a}_1(\lambda)\,\mathrm{d}\lambda,$$

$$A_2 = \int P(\lambda)\bar{a}_2(\lambda)\,\mathrm{d}\lambda,$$

$$\text{and} \qquad A_3 = \int P(\lambda)\bar{a}_3(\lambda)\,\mathrm{d}\lambda. \quad (2)$$

The chromaticity coordinates (see Sect. 4) are then $a_1 = A_1/(A_1 + A_2 + A_3)$ and $a_2 = A_2/(A_1 + A_2 + A_3)$.

Since spectral power distributions and CMFs are discrete functions, the integration in Eq. (2) is usually replaced by a sum.

### 1.2.5 Transformability of CMFs

The $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ CMFs shown in Fig. 2 are for the **RGB** (red–green–blue) primaries of 645, 526, and 444 nm. These CMFs can be linearly transformed to any other set of real primary lights, and, as illustrated in Fig. 4, to *imaginary* primary lights (i.e., physically unrealizable lights), such as the **X**, **Y**, and **Z** primaries favored by the CIE, or to the **L**-, **M**-, and **S**-cone *fundamental* primaries. Each transformation is accomplished by multiplying the CMFs by a $3 \times 3$ matrix [Eq. (4)]. See Sect. 3.2.5 of Wyszecki and Stiles [8] for more details about transformations between primaries.

The **X**, **Y**, and **Z** primaries were selected by the CIE because $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are always positive, thus simplifying computations using logarithms (which was important before computers); and, because $\bar{y}(\lambda)$ is the luminosity function (so incorporating luminosity information into the CMFs, see Sects. 5.1 and 5.6).

The three fundamental primaries (or "Grundempfindungen" – fundamental sensations) are the three imaginary primary lights that would uniquely stimulate each of the three cones to yield $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, and $\bar{s}(\lambda)$, or the **L**-, **M**-, and **S**-cone spectral

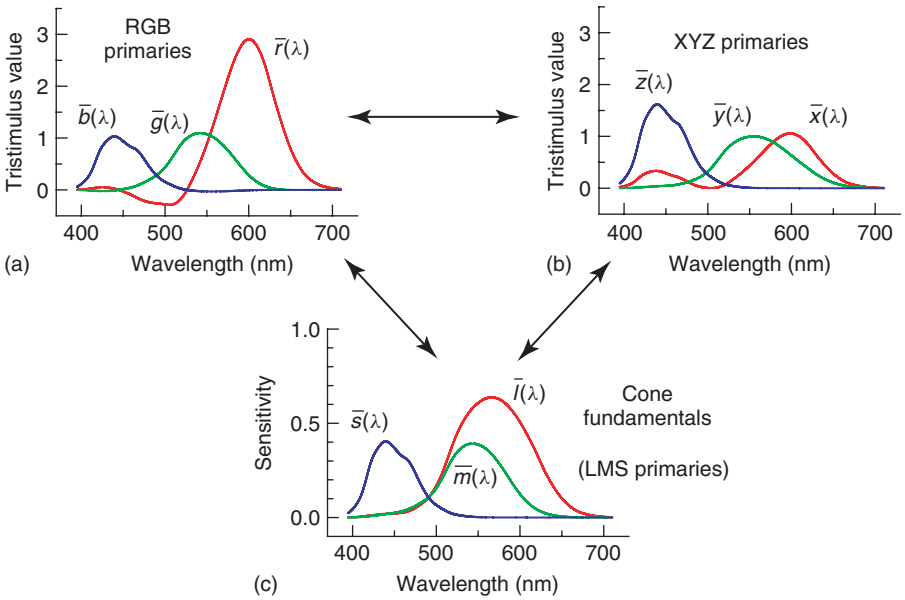


**Fig. 4** CMFs can be linearly transformed from one set of primaries to another. Illustrated here are CMFs (a) for **R**, **G**, and **B** primaries; (b) for the imaginary **X**, **Y**, and **Z** primaries; and (c) for the cone fundamental **L**, **M**, and **S** primaries (bottom). The CMFs are Judd–Vos modified CIE 1931 functions (see Sect. 5.2) and the Smith–Pokorny cone fundamentals (see Sect. 2)

sensitivity functions that underlie and determine all color matches. Although $\bar{l}(\lambda)$ and $\bar{m}(\lambda)$ cannot be obtained directly from color matches, they should, however, be a linear transformation of standard CMFs (in contrast, $\bar{s}(\lambda)$ can be determined from color matches; see Sect. 2).

### 1.2.6 Specificity of CMFs

Color-matching data are specific to the conditions under which they were measured, and strictly to the individual observers in whom they were measured. By applying the data to other conditions and using them to predict other observer's matches, some errors will inevitably be introduced.

An important consideration is the area of the retina within which the color matches were made. Standard color-matching data (see Sect. 1.2.7) have been obtained for centrally viewed fields with diameters of either 2 deg or 10 deg of visual angle. These visual angles refer to the angles subtended by objects in the external field of view at the effective optical center of the eye. The size of a circular matching field used in colorimetry is defined as the angular difference subtended at the eye between diametrically opposite points on the circumference of the field. Thus, matches are defined according to the retinal size of the matching field, *not* its physical size. A 2-deg diameter field is known as a "small" field, whereas a 10-deg one is known as a "large" field. (1 deg of visual angle is roughly equivalent to the width of the fingernail of the index finger held at arm's length.) Color matches vary with retinal size and with retinal position because of changes in macular pigment density and photopigment optical density with visual angle (see Sect. 3).

The CMFs are mean data that are also known as "standard observer" data in the sense that they are assumed to represent the color-matching behavior of a typical, standard observer. The color matches of individual observers, however, can vary substantially from these mean matches. Individual differences in lens pigment density, macular pigment density, photopigment optical density, and in the photopigments themselves can all influence color matches (see Sect. 3).

### 1.2.7 Existing CMFs

There are several existing sets of CMFs. For the central 2 deg of vision (the small-field-matching conditions), they include the CIE 1931 CMFs [11], the Judd–Vos modified 1931 CMFs [12, 13], and the Stiles & Burch [6] CMFs. For the central 10 deg of vision (the large-field-matching conditions), they include the 10-deg CMFs of Stiles & Burch [14], and the related 10-deg CIE 1964 CMFs. The CIE functions are available as $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ or $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ CMFs. The advantages and disadvantages of these CMFs are discussed in Sect. 5. The most widely used functions, the CIE 1931 CMFs, for example, are flawed at shorter wavelengths.

## 2 Cone Spectral Sensitivities

With the establishment of trichromatic color theory [15–17], a central goal in color science has been the definition of the linear transformation between $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ and the three fundamental CMFs, the three cone spectral sensitivities, $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, and $\bar{s}(\lambda)$, the first plausible estimates of which were obtained as early as 1886 [18].

When an observer matches the test and mixture fields in a color-matching experiment, the two fields are matched for each of his or her three cone types. The match, in other words, is a match *at the level of the cones*. Put more formally, it can be defined for each cone type thus:

$$\begin{aligned} \bar{l}_R\bar{r}(\lambda) + \bar{l}_G\bar{g}(\lambda) + \bar{l}_B\bar{b}(\lambda) &= \bar{l}(\lambda); \\ \bar{m}_R\bar{r}(\lambda) + \bar{m}_G\bar{g}(\lambda) + \bar{m}_B\bar{b}(\lambda) &= \bar{m}(\lambda); \\ \text{and} \quad \bar{s}_R\bar{r}(\lambda) + \bar{s}_G\bar{g}(\lambda) + \bar{s}_B\bar{b}(\lambda) &= \bar{s}(\lambda), \end{aligned} \tag{3}$$

where $\bar{l}_R$, $\bar{l}_G$, and $\bar{l}_B$ are respectively the L-cone sensitivities to the **R**, **G**, and **B** primary lights, and similarly $\bar{m}_R$, $\bar{m}_G$, and $\bar{m}_B$ are the M-cone sensitivities to the primary lights, and $\bar{s}_R$, $\bar{s}_G$, and $\bar{s}_B$ are the S-cone sensitivities. Since the S-cones are insensitive in the red, it can be assumed that $\bar{s}_R$ is effectively zero for a long-wavelength **R** primary. There are therefore eight unknowns required for the linear transformation:

$$\begin{pmatrix} \bar{l}_R & \bar{l}_G & \bar{l}_B \\ \bar{m}_R & \bar{m}_G & \bar{m}_B \\ 0 & \bar{s}_G & \bar{s}_B \end{pmatrix} \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} \bar{l}(\lambda) \\ \bar{m}(\lambda) \\ \bar{s}(\lambda) \end{pmatrix}. \tag{4}$$

Moreover, since we are often more concerned about the relative shapes of the $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, and $\bar{s}(\lambda)$, rather than their absolute values, the eight unknowns collapse to just five

$$\begin{pmatrix} \bar{l}_R/\bar{l}_B & \bar{l}_G/\bar{l}_B & 1 \\ \bar{m}_R/\bar{m}_B & \bar{m}_G/\bar{m}_B & 1 \\ 0 & \bar{s}_G/\bar{s}_B & 1 \end{pmatrix} \begin{pmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} k_l\bar{l}(\lambda) \\ k_m\bar{m}(\lambda) \\ k_s\bar{s}(\lambda) \end{pmatrix}, \tag{5}$$

where the absolute values of $k_l(1/\bar{l}_B)$, $k_m(1/\bar{m}_B)$, and $k_s(1/\bar{s}_B)$ remain unknown, but are typically chosen to scale three functions in some way so that, for example, $k_l\bar{l}(\lambda)$, $k_m\bar{m}(\lambda)$, and $k_s\bar{s}(\lambda)$ peak at unity. In the Smith–Pokorny solution

(below), $k_l\bar{l}(\lambda) + k_m\bar{m}(\lambda)$ sum to $V(\lambda)$, the luminosity function.

The unknowns in Eq. (5) can be estimated by making spectral sensitivity measurements in dichromatic observers (who lack one of the three cone types; see Sect. 3.1.5) and normal observers under special conditions that isolate the responses of single cones, or by comparing color matches made by normal and dichromatic observers. Derivations based on dichromats depend on the "loss," "reduction," or "König" hypothesis that dichromatic observers lack one of the three cone types, but retain two that are identical to their counterparts in normals [7, 19] (so that they accept all normal color matches). This assumption is now more secure, since it is possible to sequence and identify the photopigment opsin genes of normal, dichromat, and monochromat observers [20, 21], and so distinguish those individuals who conform, genetically, to the "reduction" hypothesis. The unknown value, $\bar{s}_G/\bar{s}_B$, can also be derived directly from normal color-matching data [22, 23].

Several estimates of the normal cone spectral sensitivities have been based on the loss hypothesis [23–31]. Figure 5 shows the current estimates by Smith & Pokorny [29] and Stockman & Sharpe [32].

The Smith–Pokorny estimates are a transformation of the Judd–Vos corrected CIE 1931 functions, while the Stockman–Sharpe estimates are a transformation of the Stiles & Burch 10-deg CMFs (adjusted to 2 deg). See Stockman & Sharpe (1999) for further information.

## 3
## Factors that Influence Color Matches

Several factors can influence color matches. Many of them vary between observers (differences which are referred to as *individual differences*), and so should be taken into account when trying to predict the color matches of an individual from standard CMFs. Some of them vary with retinal position, and so should be taken into account when trying to predict the color matches for retinal areas or retinal positions that differ from the centrally viewed 2-deg or 10-deg areas used to obtain the standard functions. See also Sect. 9 of the article on COLOR VISION.

### 3.1
### Individual Differences

#### 3.1.1 Lens Pigment

Light is brought into focus on the retina by the cornea and the pigmented crystalline lens. The pigment in the lens absorbs light mainly of short wavelengths (see Fig. 5b). Individual differences in lens pigment density can be large with a range of approximately ±25% of the mean density in young observers (<30 years old) [33]. Since lens density increases with the age of the observer [34, 35], the variability in the general population is even larger.

#### 3.1.2 Macular Pigment

Before reaching the photoreceptor, light must pass through the ocular media, including at the fovea, the macula lutea, which contains the macular pigment. This pigment also absorbs light mainly of short wavelengths (see Fig. 5b). Individual differences in its density can also be large, with a range of peak density from 0.0 to ca. 1.2 at 460 nm [36–38].

#### 3.1.3 Photopigment Optical Density

The axial optical density of the photopigment in the receptor outer segment varies between individuals. Decreases in photopigment optical density

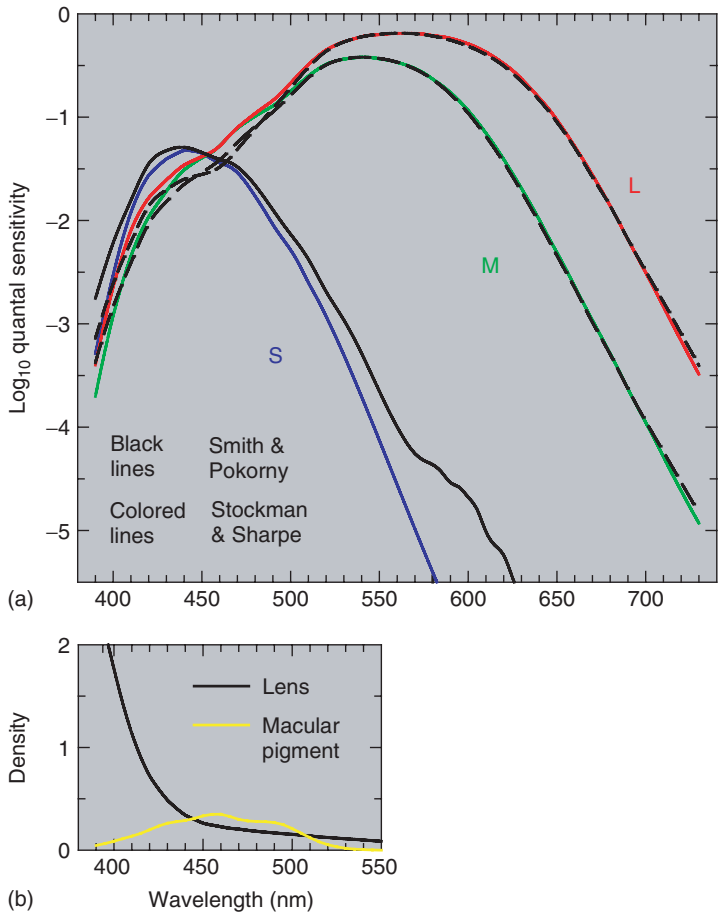


**Fig. 5** (a) **S**-, **M**-, and **L**-cone spectral sensitivity estimates of Stockman & Sharpe [32] (colored lines) compared with the estimates of Smith & Pokorny [29] (dashed lines). (b) shows the lens pigment optical density spectrum (black line) and the macular pigment optical density spectrum (yellow line) from Stockman and Sharpe [32]. Note the logarithmic vertical scale – commonly used in such plots to emphasize small sensitivities

result in a narrowing of cone spectral sensitivity curves, which cause corresponding changes to their linear transformations, the CMFs. Any corrections are most easily applied to the cone fundamentals rather than the CMFs. Estimates of photopigment optical density vary considerably depending to a large extent on the method used to estimate them, but all estimates show sizable individual differences [39–46]. Photopigment bleaching has the effect of reducing photopigment optical density (see Sect. 3.3).

### 3.1.4 Photopigment Variability

There is now clear molecular genetic and other evidence that the M- and L-cone

photopigments can vary in spectral position between observers [47], thus confirming earlier evidence for such shifts [48–52].

Large shifts away from the normal spectral position result in anomalous trichromacy (see Sect. 3.1.5). Smaller shifts occur within the normal population, because of different polymorphisms (commonly occurring allelic differences) of the M- and L-cone photopigment opsin genes. The two common polymorphic variants of the L-cone photopigment (which have either alanine or serine at position 180 of the L photopigment opsin gene) differ in spectral position by 2.7 nm or more [47].

### 3.1.5 Color Deficiency

Some color-deficient observers make dichromatic matches, since they require just two primaries to match all spectral lights. Their deficiency is therefore consistent with the lack of one of the three cone types. If their two surviving cone types are normal (see Sect. 2), they will accept the matches of normal observers. The three types of dichromacy are referred to as deuteranopia (lack of M-cones), protanopia (lack of L-cones), and tritanopia (lack of S-cones).

Other color-deficient observers make anomalous trichromatic color matches. These individuals typically have an anomalous L-cone or M-cone pigment that is shifted away from its normal spectral position. In protanomaly, the spectral position of the L-cone photopigment is shifted toward that of M, while in deuteranomaly that of the M-cone photopigment is shifted toward that of L. The size of the shift affects the severity of the deficiency.

The causes of most deficiencies now have a sound molecular genetic explanation [47]. See also Sect. 10 of the article on COLOR VISION.

## 3.2 Changes with Eccentricity

Macular pigment and photopigment optical density both decline with eccentricity. Consequently, the available CMFs and cone fundamentals, which are for centrally viewed 2- or 10-deg diameter fields, must be adjusted in order to predict accurately color matches for other viewing conditions – either for different field sizes or for different viewing angles.

One additional complication is that the S-cones are absent in approximately the central 25-min diameter of vision, so that in that region color matches become tritanopic [53].

## 3.3 Bleaching

Color matches are disturbed at high bleaching levels, because bleaching lowers the optical density of the photopigment within the photoreceptor. As noted above, this has the effect of narrowing the shapes of the cone spectral sensitivities. Corrections for the effect of bleaching can be applied directly to the cone fundamentals [54]. Corrections to other CMFs can be most easily accomplished by transforming them to cone fundamentals, correcting them, and transforming them back. Approximately half of the photopigment is bleached by a steady white light of 4.3 log photopic trolands [55].

## 3.4 Rod Influence

At mesopic levels (levels at which both rods and cones operate), trichromatic color matches can be disturbed by the influence of rods, as a result of which predicted color matches may fail [see Sect. 5.6.2 of [8]].

# 4 Color Spaces

## 4.1 Two- and Three-dimensional Color Spaces

For simplicity, color-matching data are often represented not as CMFs, but in relative units called *chromaticity coordinates*. The chromaticity coordinates ($r(\lambda)$, $g(\lambda)$, and $b(\lambda)$) of the spectrum locus (the locus of monochromatic spectral lights) are related to the CMFs ($\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$) as follows:

$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)},$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)},$$

and

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}. \quad (6)$$

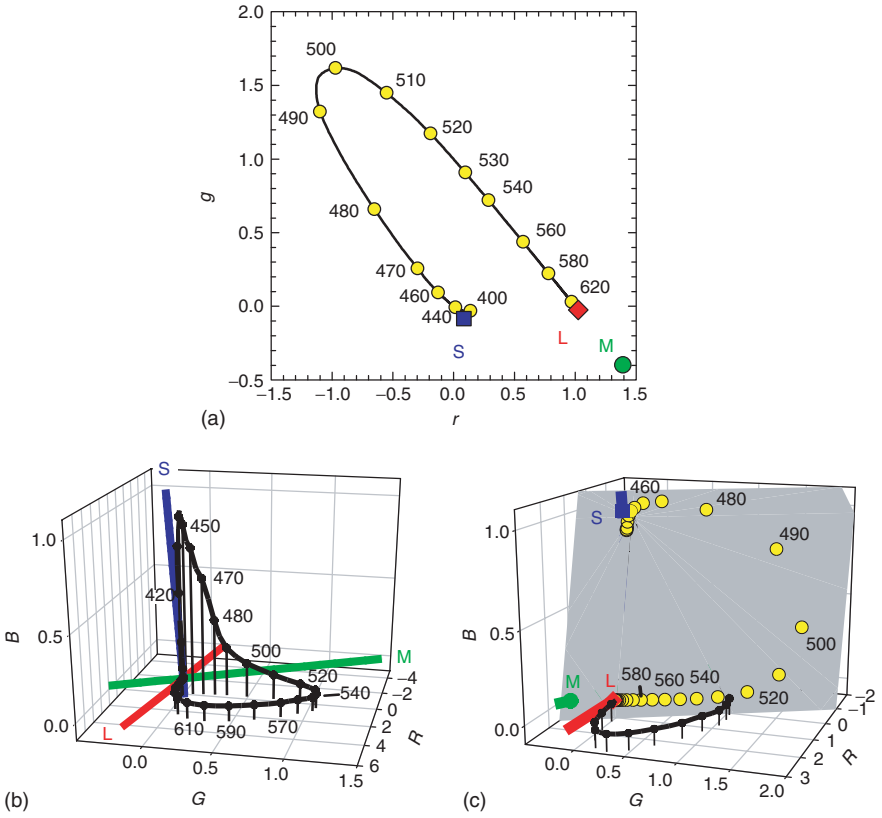


**Fig. 6** CMFs, cone fundamentals, and chromaticity coordinates. (a) Spectrum locus (continuous line) and selected wavelengths (yellow circles) plotted in the Stiles & Burch [6] 2-deg $r$, $g$ chromaticity space, and the projection of the 2-deg L- (red diamond), M- (green circle) and S- (blue square) cone fundamentals of Stockman & Sharpe [32]. (b) Spectrum locus (continuous line) and selected wavelengths (filled circles) plotted in the Stiles & Burch [6] 2-deg *RGB* space, and the L- (red line), M- (green line) and S- (blue line) cone vectors. (c) Version of CMF space with the plane $R + G + B = 1$ shown in gray. The projections of selected points on the spectrum locus onto this plane are shown by the yellow circles, and the projections of the L-, M- and S-cone vectors are shown by the red diamond, green circle, and blue square, respectively

Given that $r(\lambda) + g(\lambda) + b(\lambda) = 1$, only $r(\lambda)$ and $g(\lambda)$ are typically plotted, since $b(\lambda)$ is simply $1 - (r(\lambda) + g(\lambda))$.

Figure 6(b) shows a three-dimensional plot of the spectrum locus (continuous black line, with some wavelengths plotted as circles) in the Stiles & Burch [6] 2-deg *RGB* tristimulus space. Figure 6(a) shows the same data plotted as *r*, *g* chromaticity coordinates. Figure 7 shows the spectrum locus in the 1931 CIE *x*, *y* chromaticity space with a *very* approximate representation of the colors associated with each coordinate. While chromaticity coordinates are a convenient way of plotting spectral distributions, and predicting color mixtures, they inevitably throw away all information about intensity.

Figure 6(c) (bottom right) shows the projections of the cone vectors onto a two-dimensional *unit* plane $R + G + B = 1$ (gray plane). This plane corresponds to that of the chromaticity diagram. Estimates of the cone primaries in the Stiles & Burch 2-deg *r*, *g* space by Stockman & Sharpe [32] are plotted as the red (**L**), green (**M**) and blue (**S**) symbols in Fig. 6(a) and in the *RGB* tristimulus space as the red (**L**), green (**M**) and blue (**S**) vectors in Fig. 6(b). The projections of the fundamental primaries in the chromaticity space lie entirely outside the spectrum locus.

The counterintuitive way in which the cone primaries plot in *r*, *g* (and *x*, *y*) chromaticity diagrams provides a strong argument for using spaces, the axes of which represent cone excitations. Color spaces are more straightforward and intuitive when they are defined by the cone fundamentals and represent cone excitation. Figure 8 shows the spectrum locus plotted as *l*, *m* chromaticity coordinates (a) and as *LMS* cone excitations (b).

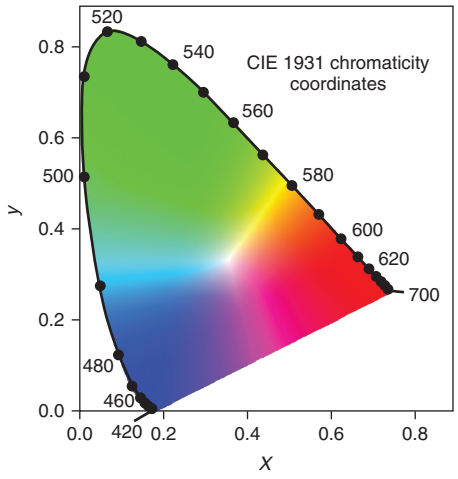


**Fig. 7** CIE (1931) *x*, *y* chromaticity space showing the spectrum locus (continuous line) and spectral wavelengths at every 10 nm (filled circles). A representation of the color of each coordinate is shown, which is necessarily *very* approximate, since it is impossible to reproduce spectral lights in print

## 4.2 Other Cone Spaces

Other transformations of the cone spaces are favored in vision research because they make clear the operation of mechanisms that work after the photoreceptors, such as color opponent mechanisms, which compute the difference between cone signals (L − M and S-[L + M]), or the luminance mechanism, which sums them (L + M).

### 4.2.1 Equal-luminance Cone Excitation Space

A common projection of the *LMS* cone space used in vision research is the MacLeod–Boynton equal-luminance plane [56, 57]. Its popularity rests, in part, on current models about postreceptoral organization, and, in particular, on the theory that only L- and M-cones contribute additively to the luminance channel so that $V(\lambda) = \bar{l}(\lambda) + \bar{m}(\lambda)$.

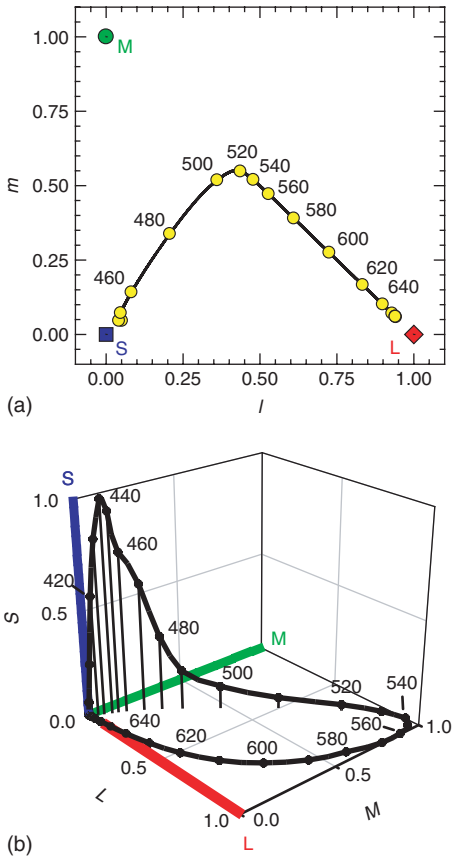


**Fig. 8** (a) Spectrum locus (continuous line) and selected wavelengths (yellow circles) plotted in the Stockman & Sharpe [32] 2-deg *l*-, *m*-cone chromaticity space. The L- (red diamond), M- (green circle), and S- (blue square) cone fundamentals plot at (1,0), (0,1), and (0,0), respectively. (b) Spectrum locus (continuous line) and selected wavelengths (filled circles) plotted in the Stockman & Sharpe [32] 2-deg *LMS* cone excitation space, and the L- (red line), M- (green line), and S- (blue line) cone vectors

The MacLeod–Boynton chromaticity coordinates are defined as

$$r_{\mathrm{MB}}(\lambda) = \frac{\bar{l}(\lambda)}{V(\lambda)},$$

$$g_{\mathrm{MB}}(\lambda) = \frac{\bar{m}(\lambda)}{V(\lambda)}$$

$$\text{and} \qquad b_{\mathrm{MB}}(\lambda) = \frac{\bar{s}(\lambda)}{V(\lambda)},$$

$$\text{where} \qquad V(\lambda) = \bar{l}(\lambda) + \bar{m}(\lambda). \quad (7)$$

Other variations of this space have been proposed [58, 59]. See also the Appendix by Brainard in Kaiser & Boynton [60]. See Fig. 4 of COLOR VISION article.

### 4.2.2 Cone Contrast Space

The cone contrast space is a scaled, nonlinear version of the cone fundamental space, in which the cone excitations produced by a stimulus are scaled separately for each cone type, according to Weber's law ($\Delta S/S$, $\Delta M/M$ and $\Delta L/L$, where $\Delta S$, $\Delta M$ and $\Delta L$ are the differential cone excitations produced by the stimulus, and $S$, $M$ and $L$ are the unchanging cone excitations produced by, for example, a background). According to Weber's Law, the incremental threshold intensity $\Delta I$ is proportional to the overall intensity, $I$, so that $\Delta I/I =$ constant. This space is useful for understanding postreceptoral visual mechanisms [61].

## 4.3 Uniform Color Spaces

The sizes and orientations of isodiscrimination ellipes plotted in the CIE 1931 *XYZ* color space vary substantially with position (such ellipses define the distances in color space that must be moved for a change in chromaticity to be discriminable). The CIE has proposed two nonlinear transformations of the CIE 1931 color coordinates to two new sets of coordinates within which the sizes of the isodiscrimination ellipses are approximately constant. The transformations are to either the CIE 1976 L*u*v (CIELUV) color coordinates or to the CIE 1976 L*a*b (CIELAB) color coordinates. Further details of these transformations can be found elsewhere [see Sect. 3.3.9 of [8]].

## 5
## Existing Color-matching Functions

The available color-matching data and functions (and as a result, the cone spectral sensitivities derived from them) vary considerably in quality. In this section, five sets of data are discussed. The most commonly used are not necessarily the best at predicting normal color matches. The luminosity function, $V(\lambda)$, which has been artificially linked to the CIE CMFs, is also discussed.

### 5.1
### CIE (1931) 2-deg CMFs

The most widely used CMFs are the CIE (1931) 2-deg CMFs [11], which are based on color-matching data obtained by Wright [4] and Guild [5]. Those data, however, are *relative* data, which give only the ratios of the three primaries required to match spectral test lights. Although sufficient for chromaticity coordinates (see Sect. 4.1), a knowledge of the absolute radiances of the matching primaries is required to generate CMFs. The CIE attempted to reconstruct this information by assuming that a linear combination of the three unknown CMFs must equal the 1924 CIE $V(\lambda)$ function [11, 62]. In addition to uncertainties about the validity of this assumption [63], the $V(\lambda)$ curve that was used is far too insensitive at short wavelengths [see Fig. 13 of [54]].

### 5.2
### Judd, Vos-modified CIE 2-deg CMFs

In an attempt to correct the CIE 1924 $V(\lambda)$ function, Judd [12] increased the sensitivity of $V(\lambda)$ below 460 nm, and derived a new set of CMFs also based on the Wright and Guild data (see Wyszecki & Stiles [8] Table 1 (5.5.2)), which were later slightly modified by Vos [13, Table 1]. These CMFs are in common use in vision research as their transformation, the Smith–Pokorny cone fundamentals (see Sect. 2).

The substantial modifications to the $V(\lambda)$ function introduced by Judd had the unwanted effect of producing CMFs that are relatively insensitive near 460 nm (where they were unchanged). Although this insensitivity can be roughly characterized as being consistent with a high macular pigment density [6, 32, 64] (see also Sect. 3.1.2), the CMFs are somewhat artificial and thus removed from real color matches.

### 5.3
### Stiles & Burch (1955) 2-deg CMFs

The assumption that $V(\lambda)$ is a linear combination of the CMFs is entirely unnecessary, since CMFs can be measured without any recourse to photometric data. The Stiles & Burch [6] 2-deg CMFs are an example of directly measured functions. Though referred to by Stiles as ''pilot'' data, these CMFs are the most extensive set of directly measured color-matching data for 2-deg vision available, being averaged from matches made by ten observers. Even compared in relative terms, there are real differences between the CIE 1931 and the Stiles & Burch [6] 2-deg color-matching data in the range between 430 and 490 nm [see Fig. 1 of [6]]. These CMFs are seldom used.

### 5.4
### Stiles & Burch (1959) 10-deg CMFs

The most comprehensive set of color-matching data are the large-field, centrally viewed 10-deg CMFs of Stiles & Burch [14]. Measured in 49 subjects from approximately 390 to 730 nm (and in 9 subjects

from 730 to 830 nm), these data are probably the most secure set of existing CMFs. Like the Stiles & Burch [6] 2-deg functions, the 10-deg functions represent directly measured CMFs, and so do not depend on measures of $V(\lambda)$. These CMFs are the basis of the Stockman and Sharpe [32] cone fundamentals (see Sect. 2).

### 5.5 CIE (1964) 10-deg CMFs

The large field CIE 1964 CMFs are based mainly on the 10-deg CMFs of Stiles & Burch [14], and to a lesser extent on the arguably inferior and possibly rod-contaminated 10-deg CMFs of Speranskaya [65]. While the CIE 1964 CMFs are similar to the 10-deg CMFs of Stiles & Burch (1959), they differ in several ways that compromise their use as the basis for cone fundamentals [32].

### 5.6 Luminance and the Luminosity Function, $V(\lambda)$

The CIE combined luminosity functions for 2-deg [$V(\lambda)$ or $\bar{y}(\lambda)$] or 10-deg [($V_{10}$ ($\lambda$) or $\bar{y}_{10}(\lambda)$] vision with color-matching data by making one of the CMFs equal to the luminosity function (see above). It should be recognized, however, that luminosity functions are quite distinct from CMFs and cone fundamentals. In particular, the shapes of luminosity functions *change with chromatic adaptation* [66, 67], whereas the CMFs and cone spectral sensitivities do not (until bleaching levels). Biologically, the cone spectral sensitivities are receptoral, whereas the luminosity function is postreceptoral.

$V(\lambda)$ is a photometric measure of luminous efficiency or spectral sensitivity that is defined as the effectiveness of lights of different wavelength in specific photometric matching tasks, which today are usually heterochromatic flicker photometry (HFP), in which rapidly alternating lights are matched in intensity to eliminate the perception of flicker, or a version of side-by-side matching, in which the relative intensities of the two half fields are set so that the border between them appears "minimally distinct" (MDB). These tasks are favored because they produce, in contrast with some of the methods used to obtain the 1924 $V(\lambda)$, additive results [63, 68]. It should be noted that luminance does not define how bright things actually appear. Fields that are equal in luminance often differ substantially in apparent brightness.

There is no *a priori* reason why colorimetric data should depend on photometric data, as was the case for the CIE 1931 functions. Given modern calibration methods, there is no justification for using photometric data to alter or adjust colorimetric data.

## 6 Conclusion

Colorimetry, the numerical specification of the color of visual stimuli, is related to the spectral sensitivities of the three cone photoreceptors. Colorimetry is more intuitive when defined in terms of cone excitations than when defined in terms of imaginary primaries, such as the CIE **XYZ** primaries. In the past, cone spectral sensitivities were too uncertain to form the basis of a colorimetric system, but that is no longer the case.

Most of the CMFs, chromaticity coordinates of the spectrum locus, cone fundamentals, and other functions described here can be downloaded from: http://www.cvrl.org

Color-matching data and CMFs tell us which spectral distributions will match under a given set of viewing conditions for a given observer. However, they tell us little about the actual *color appearance* of the match, which can vary enormously with the viewing conditions.


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## Glossary

**2 deg:** Shorthand for the "small-field" color matches made with centrally viewed, circular fields subtending 2-deg diameter of visual angle.

**10 deg:** Shorthand for the "large-field" color matches made with centrally viewed, circular fields subtending 10-deg diameter of visual angle.

**Chromaticity Coordinates:** $x$, $y$, which in terms of the tristimulus values are $X/(X + Y + Z)$ and $Y/(X + Y + Z)$, respectively (or $r$, $g$ for $RGB$ space, or $l$, $m$ for $LMS$ space).

**CIE:** Commission Internationale de l'Éclairage or International Commission on Illumination. An organization, a goal of which is to generate international standards in color and lighting.

**Color Match:** A subjective match between two lights of different spectral power distributions (which are therefore metamers).

**Color-matching Functions:** $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. Tristimulus values of the equal-energy spectrum locus.

**Colorimetry:** The measurement and numerical specification of color.

**Cone Fundamentals:** Cone spectral sensitivities. $\bar{l}(\lambda)$, $\bar{m}(\lambda)$, and $\bar{s}(\lambda)$ in CMF notation. These are the color-matching functions that would result if primaries that uniquely stimulated the three cones could be used.

**Large-field Matches:** Color matches for centrally viewed, circular fields subtending 10-deg diameter of visual angle.

**Metamers:** Two lights that match, but are physically different. An example is the match between a spectral yellow and a mixture of spectral red and green (see Fig. 1).

**Photometry:** The measurement and numerical specification of the luminous efficiency of lights. Intended to be independent of the chromaticities of the lights.

**Photopic Luminosity Function:** Photometric measure of luminance efficiency as a function of wavelength under photopic (i.e., rod-free) conditions: $V(\lambda)$ or $\bar{y}(\lambda)$.

**Photoreceptors:** The light-sensitive receptors that lie on the rear surface of the eye that transduce photons into electrical signals. They can be sensitive, nighttime rods or relatively insensitive, daytime cones.

**Primary lights: e.g., R, G, B:** The three independent primaries (real or imaginary) to which the test light is matched (actually or hypothetically). They must be independent

in the sense that no combination of two can match the third.

**Small-field Matches:** Color matches for centrally viewed, circular fields subtending 2-deg diameter of visual angle.

**Standard Observer:** The standard observer is the hypothetical person whose color-matching behavior is represented by a particular set of mean CMFs.

**Trichromacy:** The ability of normal observers to match test lights with a mixture of three independent primary lights, one of which invariably has to be added to the test light to complete the match.

**Tristimulus Values:** *R*, *G*, *B*, the amounts of the three primaries required to match a given stimulus.

**Univariance:** The output of a photoreceptor varies *unidimensionally* only according to the rate of photon absorption.

**Visual Angle:** The angle subtended by an object in the external field at the effective optical center of the eye.

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